New Results for the Packing Equal Circles in a Square Problem*

L. G. Casado and I. García
Department of Computer Architecture and Electronics
Almería University, 04120, Almería, Spain
leo@miro.ualm.es, inma@monet.ualm.es

Péter Gábor Szabó
Department of Applied Informatics
József Attila University, 6701, Szeged, Hungary
pszabo@inf.u-szeged.hu

The packing circles in a square problem can be formulated by several equivalent ways. We used the following: "Locate n points in a unit square, such that the minimum distance m_n between any two of them is maximal".

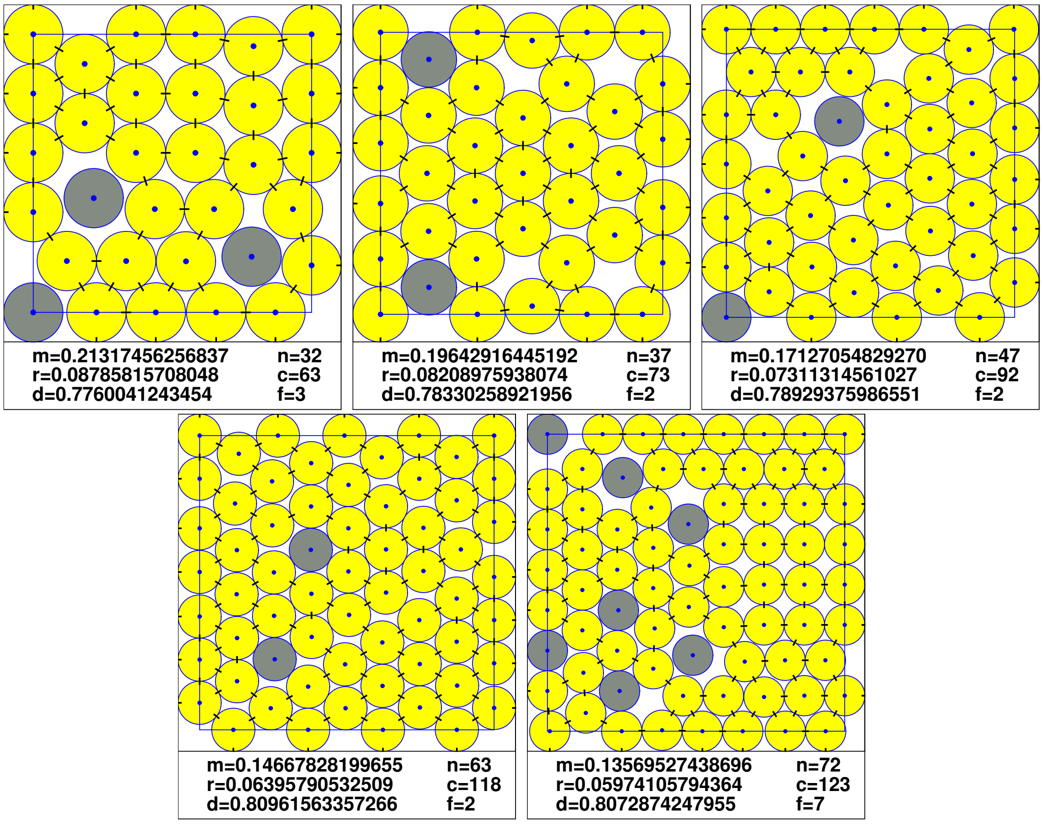
Use deterministic methods is preferable [1]. Deterministic methods ensure that their solutions are optimal. Optimal solutions for $n \leq 27$ are now possible to be known. The main problem of deterministic methods is that as the number of spread points increased, an explosion of the computational burden arises. For instance, for $n = 7$ the number of times that a program must be run is 8 but for $n = 14$ is $9,808$ and for $n = 23$ is $288,873,270$ (see http://saturn.tcs.hut.fi/pub/packings/square/). In these algorithms it is also necessary to know a good lower bound of the solution. So, stochastic algorithms are very useful not only to find a good packings but also to provide lower bounds of the optimal solutions.

In this work a stochastic global optimization algorithm, called TAMSASS-PECS (Threshold Accepting Modified Single Stochastic Search for Packing Equal Circles in a Square), has been designed. TAMSASS-PECS algorithm is based on the Threshold Accepting method [3] and on our modified version of SASS [4, 5, 6, 7, 8] (MSASS). TAMSASS-PECS is an algorithm which formally is very similar to the Simulating Annealing algorithm. TAMSASS-PECS sets up and updates parameters for the MSASS procedure which is iteratively executed until stopping criterion is reached. MSASS is in charge of perturbing the current location of a point i (s_i). This perturbation is intended to increase the minimum value of the distances ($d_{i,j}$) between i and any point j ($1 \leq j \neq i \leq n$). It moves the point i from s_i to a new location s'_i, and computes the value of the minimum distance $d'_{i,j}$. Following the Threshold Accepting strategy, a move is accepted if $d'_{i,j} > d_{i,j}T_h$, where T_h is the threshold level. New trial locations of point i, s'_i, are restricted to the neighborhood of the current location of the point i, s_i. This neighborhood is determined by a normal distribution $N(0, \sigma\mathbf{I})$. While the Threshold Accepting condition is not satisfied, new locations for the point i are tested following the classical SASS algorithm, although the number of trials are subject to a maximum value.

As all local search algorithms, the probability to find a global solution grows with the number of executions. Results for $n = 2, ..., 100$ have been obtained running the algorithm several times. For cases n =32, 37, 47, 63 and 72, better results than those shown in the literature were found. The figure below shows these packings.

In figure, m is the maximal minimum distance between two centers of the circles, r is the radius of the circles in the unit square, d is the density, n is the number of circles, c is the number of contacts between circles and between circles and edges (depicted as little lines) and f is the number of free circles (depicted in dark grey). Other results of the algorithm can be found in the home page of Péter Gábor

*This work was supported by SOCRATES-ERASMUS program (25/ERMOB/1998-99),by the Ministry of Education of Spain(CICYT TIC96-1125-C03-03) and by the Consejería de Educación de la Junta de Andalucía (07/FSC/MDM).



Szabó (http://www.inf.u-szeged.hu/∼pszabo).

References

[1] R. Peikert, D. Würtz, M. Monagan, and C. de Groot. Packing circles in a square: a review and new results. In P. Kall, editor, *System Modelling and Optimization: Proceedings of the 15th IFIP Conference, Zürich, Switzerland, September 2-6, 1991*, volume 180 of *Lecture Notes in Control and Information Sciences*, pages 45–54. Springer-Verlag, Berlin, 1992.

[2] K.J. Nurmela and P.R.J. Östergård. Packing up to 50 equal circles in a square. *Discrete & Computational Geometry*, 18:111–120, 1997.

[3] G. Dueck and T. Scheuer. Threshold accepting: a general purpose optimization algorithm appearing superior to simulating annealing. *J. Comput. Phys.*, 90:161–175, 1990.

[4] D.C. Karnop. Random search techniques for optimization problems. *Automatica*, 1:111–121, 1963.

[5] J. Matyas. Random optimization. *Automatization and Remote Control*, 26:244–251, 1965.

[6] J.R. McDonnell and D. Waagen. Evolving recurrent perceptrons for time-series modeling. *IEEE Trans. on Neural Networks*, 5(1):24–38, 1994.

[7] S.S. Rao. *Optimization Theory and Applications.* John Willey and Sons, New York, 1978.

[8] F.J. Solis and J.B. Wets. Minimization by random search techniques. *Math. of Operations Research*, 6(1):19–50, 1981.