# An Interval Branch and Bound Method Dedicated to the Optimal Design of Piezoelectric Actuators

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*Abstract:* - The purpose of this article is to show the advantage of a deterministic global optimization method in the optimal design of electromechanical actuators. The numerical methods classically used are found either on Non-linear Programming techniques (Augmented Lagrangian, Sequential Quadratic Programming, ...) or on stochastic approaches which are more satisfactorily adapted to global optimum research (Genetic Algorithm, Simulated Annealing, ...). However, these kinds of methods only guarantee to reach the global optimum with some probabilities. The present paper proposes a deterministic interval Branch and Bound algorithm adapted to the dimensioning of a heterogeneous bimorph bar, working in bending quasi-static mode. This problem is formulated as a optimization problem with constraints and mixed variables: real and categorical types. Original solutions are given and discussed at the end of the paper.

*Keywords:* - Interval Analysis, Deterministic Global Optimization, Branch and Bound algorithms, Design of Piezoelectric Actuators, Mixed Problem, Categorical Variables.

## 1 Introduction

The design of electromechanical actuators calls into play a great number of parameters which are subjected to different physical laws. By doing an approximation of these physical laws, the structure dimensioning relations can be expressed analytically. Thus, the problem of the optimal design of an electrical or electromechanical actuator can be reformulated as an optimization problem [10], [6].

Since the last years, several global optimization methods were introduced to solve these particular optimization problems which are generally non-linear and non-convex with mixed variables (real and integral), for example genetic algorithms, simulated annealing, Monté Carlo Method, ...

Our choice was to study a global deterministic approach to be sure that the global solution will be found at the end of the algorithm, like interval Newton algorithm [3], interval Branch and Bound method [12], [5], and DC-algorithms [4]...

Our first practical result was about the dimensionning of slotless permanent magnet machine [7]. The problem was a mixed constrained multimodal optimization problem which cannot permit to find the global solutions by using classical Augmented Lagrangian Methods, like in [6]. Then, in this case, our deterministic global optimization algorithms [9] were particularly efficient.

In this article, the optimal dimensioning of a piezoelectric actuator: a heterogeneous bimorph bar working in bending quasi-static mode, is carried out by the extension of classical interval Branch and Bound algorithms [12], [9]. In a previous work [10], an analytical model of the bimorph was established on the basis of classically allowed assumptions in theory of the beams and piezoelectricity, the field of validity of this model was specified then by comparison with a program using finite elements and the obtained results were validated and refined using measurements practiced on a model of bimorph.

The considered analytical model presents some new difficulties to deal with the inverse problem of the conception: which material will produce the optimum ?

Thus, the optimization problem can be formulated as a mixed (real and categorical variables) constrained optimization problem:

$$\begin{aligned} \max_{(x,k)\in X\times K} \quad & f(x,k) \\ with \quad & h_i(x,k) = 0, \forall i \in \{1,\cdots,n_h\} \\ & g_i(x,k) \le 0, \forall i \in \{1,\cdots,n_g\} \end{aligned} \tag{1}$$

Where $X$ is a vector of real compact intervals (an hypercube) and $K$ is a set of $m$ distinct enumerate sets which will describe the possible structure of our considered piezoelectric bimorph.

The main difficulty is: how can we deal with the categorical variables $k$? One can combine all the possibilities, but if one has a lot of categorical variables, it becomes very hard to carry out this; or one can extend classical interval Branch and Bound algorithms to deal with the categorical variables.

# 2 Extension of Interval Branch and Bound Algorithms

The main adavantage of these kinds of methods are that they permit to enclose with accuracy the global optimum and also all the solution points. The efficiency of these algorithms based on interval analysis had already been validated in a lot of domain, like Chemical Process [8] and also Electromagnetism for the design of a slotless permanent magnet machine [7]. In this paper, the algorithm is improved by considering the constraints, and it is modified to deal with the fact that some variables are not real but categorical variables : variable of type, for example steel or brass. Of course, they could be represented by integer values but the ordering relation are not kept.

For the detailed algorithms of Branch and Bound, please see [9], [12] and [5].

## 2.1 Branch and Bound Principle

These kinds of algorithm are based on a exclusion principle. Thus, each step tries to discard some parts of the initial domain where the solution is searched.

The main idea is to subdivide the initial domain $X \subseteq \mathbb{R}^n$ into smallest sub-boxes $Z \subseteq X$ and to delete the considered box $Z$, if and only if one can prove that:

1. $Z$ cannot contain the global optimum,
2. At least, one constraint cannot be satisfied for all elements of $Z$.

To proceed these eliminations, one just needs the computations of lower or upper bounds of a given function $f$ over a box $Z$, let us denote it $lb(f,Z)$, respectively $ub(f,z)$.

If $\tilde{f}$ denotes the current solution (in fact, it is just the better evaluation of $f$ found at this step of the algorithm, and all the constraints are satisfied), and $h$ the functions of equality constraints and $g$ the functions of inequality constraints, like in (1) then a considered sub-box $Z \subseteq X \subset \mathbb{R}^n$ can be deleted if and only if:

1. No global solution in $Z$: $lb(f,Z) > \tilde{f}$, a lower bound of $f$ over $Z$ is greater than a solution already found, then none point in $Z$ can be a global minimum (reciprocally $ub(f,Z) < \tilde{f}$ for maximization problems),
2. Unsatisfied inequality constraint: it exists $k$ such that $lb(g_k,Z) > 0$,
3. Unsatisfied equality constraint: it exists $k$ such that $lb(h_k,Z) > 0$ or $ub(h_k,Z) < 0$ (in this case, one must take an $\varepsilon$ very small).

Thus, at the end of the algorithm, one expects to have a good enclosure of the global minimum and all the global optimizers, because for all the sub-boxes $Z \subseteq X$ which cannot be deleted during the execution:

1. $lb(f,Z) \le \tilde{f}$,
2. It is impossible to prove that one constraint is unsatisfied.

As one stops when the global minimum is sufficiently accurate: $\tilde{f} - \min_Z lb(f,Z) \le \varepsilon$ and also when all the sub-boxes $Z$ are sufficiently small, one global solution is given by the current solution $\tilde{f}$ and also by all the remaining sub-boxes.

There exists a lot of methods to compute bounds of a function $f$ over a box $X$. Here, one chooses interval analysis.

## 2.2 Basis of interval arithmetic

Interval arithmetic was introduced by Moore [11] as a basic tool for the estimation of numerical errors in machine computations. Instead approximating the real value $x$ by a machine representable number, a pair of machine representable numbers is used what defines an interval enclosing $x$.

Let $I\!I$ be the set of real compact intervals $[a, b]$, where $a$, $b$ are real numbers. The basic arithmetic operations for intervals are defined as follows:

$$\begin{cases} [a,b]+[c,d]=[a+c,b+d] \\ [a,b]-[c,d]=[a-d,b-c] \\ [a,b]\times[c,d]=[\min\{a\times c, a\times d, b\times c, b\times d\}, \\ \quad \max\{a\times c, a\times d, b\times c, b\times d\}] \\ [a,b]\div[c,d]=[a,b]\times[\frac{1}{d},\frac{1}{c}] \text{ if } 0\notin[c,d] \end{cases} \tag{2}$$

The above definitions (2) show that subtraction and division in $I\!I$ are not the inverse operations of addition and multiplication, respectively contrary to the real case. Unfortunately, the interval arithmetic doesn't keep all the the properties of the standard one.

In the above rules of interval arithmetic, the division by an interval containing zero is undefined. But it is often useful to remove this restriction. The resulting arithmetic then is called *extended interval arithmetic*, see [3], [12].

**Notations -**

If $A$ is an element of $I\!I$ then we also write $A = [a^L, a^U]$ denoting the lower and the upper boundaries of $A$ by $a^L$ and $a^U$.

The width of an interval $A$ is denoted by $\mathrm{w}(A) = a^U - a^L$ and the midpoint by $\mathrm{mid}\ (A) = \dfrac{a^L + a^U}{2}$.

The set of n-dimensional interval column vectors is denoted by $I\!I^n$.

If $A = (A_1, \cdots, A_n)^T$ is an element of $I\!I^n$, the width of $A$ is defined to be $\mathrm{w}(A) = \max\{\mathrm{w}(A_i) : i = 1, \cdots, n\}$ and the midpoint of $A$ to be $\mathrm{mid}\ (A) = (\mathrm{mid}\ (A_1), \cdots, \mathrm{mid}\ (A_n))^T$.

For all others details about interval arithmetic, see [11], [3], [12].

## 2.3 Inclusion function

**Definition 1** *An inclusion function $F$ is an interval function: $F : I\!I^n \longrightarrow I\!I$ such as it encloses the range of $f$ over all boxes $X$ in $I\!I^n$. Therefore, $f(X) = [\min_{x\in X} f(x), \max_{x\in X} f(x)] \subseteq F(X)$.*

**Theorem 1** *The* natural extension *of an expression of $f$ into interval, consists by replacing each occurrence of a variable by it corresponding interval (which encloses it), and then by applying the above rules of interval arithmetic. Special procedures for bounding trigonometric and transcendental functions allow the extension of this procedure to a great number of analytical functions.*

*This interval function, so constructed, is an inclusion function of $f$, let us denote it by $F(X) = [F^L(X), F^U(X)]$.*

The demonstration is given in [12].

The bounds so evaluated by the natural extension of an expression of $f$ are not always accurate, and often one proposes several other interval techniques to compute lower or upper bounds, please see [9], [3], [11], [12] for a thorough survey and discussion on this point.

For our considering design problem, it is not useful to implement other techniques: the natural extension into interval is sufficient.

## 2.4 Extensions to the case of categorical variables

In the considered problem some variables are real values and others are categorical ones. Thus, to deal with the real variables, the interval representation and the interval arithmetic can directly be used; for the categorical variables, one must introduce a new representation and a new method to subdivide this kind of enumerate set. The main idea is to construct a new inclusion function defined over the two distinct sets: interval set and enumerate set.

In the formulation problem (1), all the functions $f$, $g_i$ and $h_j$ provide results in the real set. Thus, one wants to extend this, to enclose (by an interval) the range of these functions over a considered box.

In fact, the categorical variables $k_i$ just permit to determine the value of the parameters of a material; for example the Young modulus which depends only about the used material: ceramic P1-89 or steel or brass... These real parameters belong to the expression of the considered functions, and therefore one must find the resulting interval which will enclose these real parameters if one takes into account a part of an enumerate set.

Thus, there are several univariate real functions which depend of only one categorical variable, let's denote them by $a_i : K_j \longrightarrow \mathbb{R}$. One wants to construct a method to enclose the range of the function $a_i$ over a subset $O$ of the enumerate set $K_j$ (component of the set $K$, (1)), let us denote this inclusion function $A_i$, and $A_i(O)$ provides a real interval.

1. Enumerating method: Let us consider $O_j \subseteq K_j$, therefore

   $$A_i(O_j) \longleftarrow \quad [\min\{a_i(k) : \forall k \in O_j\}, \max\{a_i(k) : \forall k \in O_j\}]$$

   The problem is: at each step of the algorithm, one must enumerate all the possibilities to find the right lower and upper bounds.

   **Remark 1** *In that case, $A_i(O_j) = a_i(O_j)$, because one has the real bounds.*

   **Lemme 1** *The order of the complexity of the procedure which computes the minimum and the maximum is about card$(O_j)$. That is not so hard. Nevertheless, it must be done every time for all the functions and for all the enumerate set $K_j$. In fact, it depends of the total number of functions which is fixed; let's denote it by $n_a$ and the complexity of the complete procedure to compute all the bounds for all the $n_a$ functions is at the order of $n_a \times \max_j card(O_j)$*

2. Heuristic enumerating method:

   If the added complexity is too important (if one has a lot of enumerate sets), one proposes to introduce a heuristic to discard some uninteresting computations.

   At the first step of the algorithm, one calculates:

   $$A_i(K_j) \longleftarrow \quad [\min\{a_i(k) : \forall k \in K_j\}, \max\{a_i(k) : \forall k \in K_j\}]$$

   and then by considering an enumerate set $O_j \subseteq K_j$:

   - If card$(O_j) < n_O$, with $n_O$ a small integer value (=4) Then Compute the real bounds:

     $$A_i(O_j) \longleftarrow \quad [\min\{a_i(k) : \forall k \in O_j\}, \max\{a_i(k) : \forall k \in O_j\}]$$

   - Else Return $A_i(K_j)$ already computed.

   The problem is to fix the parameter $n_O$ to significantly improve the total CPU-time and so, to validate the use of this heuristic.

   **Remark 2** *It is clear that this heuristic got an interest if and only if the number of enumerate sets and their numbers of elements are very important.*

3. Sorting method: In this method, one prefers to sort the enumerate sets to simplify the computations of the bounds for some functions.

   - First, one sorts in the increased order the two sets $\{a_1(k) : k \in K_j\}$ and the corresponding set $K_j$, that one notices also $K_j$: the enumerate sorted set for the first considered function $a_1$.
   - By considering a subset $O$ of $K_j$ then

     $$A_1(O) \longleftarrow [a_1(\text{first}(O)), a_1(\text{last}(O))]$$

     Where first$(O)$ provides the first element of the sorted set $O$ and last$(O)$ the last one.

     It is feasible if and only if the enumerate set $O$ is also sorted: it is constructed directly by subdividing $K_j$.

     Nevertheless, it is possible that this sorted set doesn't correspond to some other functions $a_i, i \geq 2$, then one uses one of the two previous methods.
   - Another idea is to sort at the first step, the same enumerate set $K_j$ for all the functions $a_i$, let us noticed them $K_j^i$, with $K_j = K_j^1$.

     Furthermore, by keeping these scheduled sets $K_j^i$ and the previous places of the minimum $pm_{a_i}$ and the maximum $pM_{a_i}$ of all the considered functions $a_i$ over a previous subset $O \subseteq K_j$, one just needs to recompute two new bounds for each $a_i$ at the following step when $O$ will be subdivided.

     If one considers that the subdivision of $O$ gives two new enumerate sets $O_1$ and $O_2$, such as $O = O_1 \cup O_2$ and $O_1 \cap O_2 = \emptyset$, that involves:

- if the element of $O$ in place $pm_{a_i}$ is in $O_1$ then one only recomputes the place $pm_{a_i}$ for the second subset $O_2$ else one only recomputes the place $pm_{a_i}$ for $O_1$,
- if the element of $O$ in place $pM_{a_i}$ is in $O_1$ then one only recomputes the place $pM_{a_i}$ for the second subset $O_2$ else one only recomputes the place $pM_{a_i}$ for $O_1$.

Let's remark that it is very simple to compute the new place of the minimum or of the maximum because the enumerate set $K_j^i$ is looked through, until an element belongs to $O_1$ (resp. $O_2$): that's the new minimum (resp. maximum).

By using these techniques, one must store the considered subsets $O_j$ of $K_j$, and all the optimal positions for each function $a_i$ (or directly the values of the bounds) of $a_i$ over $O_j$, that involves the introduction of a new (abstract) structure; all the sorted sets $K_j^i$ were only stored at the first steps of the algorithm.

Of course, one carries out all these procedures for all $j \in \{1, \cdots, \mathrm{card}(K)\}$ and for all the corresponding functions $a_i$.

Thus, one proposes here some techniques to deal with the categorical variables. Inclusion functions $A_i$ were constructed and it is sure that they enclose the range of the real univariate function $a_i$ over a considered enumerate subset $O$, this for all the sets $K_j$. Then, new inclusion functions were construted to deal with mixed variables.

# 3 Optimal Dimensioning of a Bimorph

In this section, the algorithm above is used for the design of a piezoelectric actuator: a bimorph. His analytical modeling was previously developed and published in [10]. A priori, the interest of our method could be evident on this example, because the criterion and the constraint equations are non-linear and non-convex, furthermore two variables are categorical variables. Therefore, some properties are lost (like the differentiability). Our approach is illustrated through a dimensioning problem of a bimorph used in a magnetic metrology device with "vibrating sample", see Figure 1.

## 3.1 Considered specifications

The problem treated relates to the dimensioning of an actuator of a vibrating sample magnetometer (VSM), device making it possible to experimentally determine the hysteresis loops and the magnetic characteristics of materials by an extraction method [1]. The sample, placed in a uniform magnetic field with variable intensity created by Halbach cylinders [2], is driven in vertical vibration by the intermediary of an interdependent rod and actuator. The magnetic moment of the sample can then be deduced from the induced electromotive forces at the boundaries of pickup coils. The critical aspects of the specifications lie in the fact that the actuator used must be non-magnetic, space-saving and created vibrations should not be transmitted to the pickup coils located near the sample. Thus, one solution considered consists in using two piezoelectric bimorphs working in parallel, the free end of each being connected by a mechanical part in order to drive the sample to be characterized vertically as illustrated by Figure 1.

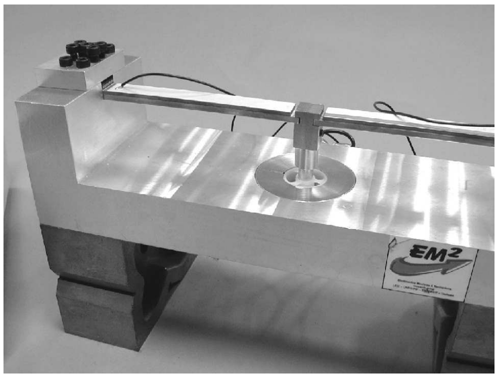

Figure 1: Vibrator for magnetometer build with two piezoelectric bimorphs (Photo LEEI).

Each of the two bimorphs has to be dimensioned must satisfy the following specifications:

- the peak-to-peak amplitude of the movement is fixed at 80 $\mu$m, that involves 40 $\mu$m for the displacement $\delta$,

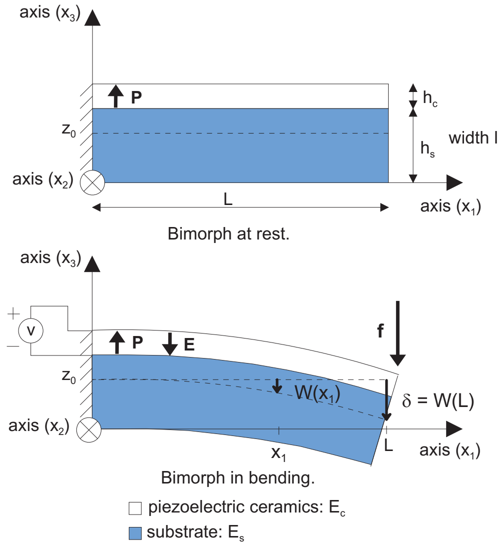


Figure 2: Diagrams of the bimorph at rest and in bending.

- the frequency of the use, can vary between 10 and 100 Hz,
- the mass of the set "rod-sample" is about $40 \times 10^{-3}$ kg, which corresponds to an applied force $f = 0.4$ N,
- the maximum amplitude of the voltage delivered by the supply is limited to 250 V,
- the electric field $E_3$ within ceramic is fixed at 100 V/mm.

## 3.2 Considered Modelization

The considered bimorph consists of a layer of piezoelectric ceramic, thickness $h_c$, and of a layer of passive material (for example steel), here called substrate, thickness $h_s$. Ceramic and substrate are stuck together. In this connection, the working assumption retained is that the assembly is perfect. The bar is length $L$, width $l$, see Figure 2.

The vertical displacement $\delta$ is given by the following equation:

$$\delta = \frac{3L^2\beta\eta(1+\eta)d_{31}E_3}{h_s(1+4\beta\eta+6\beta\eta^2+4\beta\eta^3+\beta^2\eta^4)} + \frac{4L^3(1+\beta\eta)f}{lE_s h_s^3(1+4\beta\eta+6\beta\eta^2+4\beta\eta^3+\beta^2\eta^4)} \quad (3)$$

To simplify the notation, let us denote $\beta = \frac{E_c}{E_s}$ and $\eta = \frac{h_c}{h_s}$. Where $E_c$ and $E_s$ are respectively the Young modulus of the ceramic and the Young modulus of the substrate. In fact, $d_{31}$, $E_c$ and $E_s$ are real univariate function depending on some categorical variables: the type of ceramic or the type of substrate.

The developed method of Global Optimization is validated on the design problem formulated as follows:

$$\max_{h_c,h_s,L,l,k_c,k_s} \delta(h_c,h_s,L,l,k_c,k_s)$$
$$\text{with:} \quad \left| \begin{array}{lcl} h_c & \in & [0.1,\ 5]\,10^{-3}\ \mathrm{m} \\ h_s & \in & [3,\ 20]\,10^{-3}\ \mathrm{m} \\ L & \in & [90,\ 100]\,10^{-3}\ \mathrm{m} \\ l & \in & [7,\ 20]\,10^{-3}\ \mathrm{m} \\ k_c & \in & \{P1-89,\ P1-90\} \\ k_s & \in & \{\mathrm{steel},\ \mathrm{brass}\} \end{array} \right. \quad (4)$$
$$\text{subject to:} \quad \left\{ \begin{array}{rcl} 3(h_c+h_s) & \leq & L \\ h_c E_3 & \leq & 250 \\ l h_c L 10^6 & = & 2.4 \end{array} \right.$$

The used variables $h_c$, $h_s$, $L$ and $l$, correspond to the bimorph dimensions; there are three constraints: the first limits the thickness of the bender compared to its length, the second relates the maximum authorized voltage and the last limits the volume of ceramic.

The two last variables $k_c$ and $k_r$ are categorical variable and so they determine the values of $d_{31}$, $E_c$ and $E_s$, as follows:

- if $k_c = P1-89$ then $d_{31} = -108 \times 10^{-12}$C/N and $E_c = 9.38 \times 10^{10}$N/m$^2$,
- if $k_c = P1-91$ then $d_{31} = -247 \times 10^{-12}$C/N and $E_c = 5.98 \times 10^{10}$N/m$^2$,
- if $k_s$ =steel then $E_s = 2.1 \times 10^{11}$N/m$^2$,
- if $k_s$ =brass then $E_s = 1.05 \times 10^{11}$N/m$^2$.

## 3.3 Optimal Dimensionning Results

The resolution of the optimization problem formulated previously (4) is carried out for each of the four possible configurations of bimorphs, see all the numerical results in Table 1. Nevertheless, the problem can be directly solved by our new extended algorithm which deals with the categorical variables. In that case, the last column of the Table 1 is directly obtained, in incredible short CPU-times: 4 seconds on a Hewlett Packard computer (180MHz) and less than 1 second on a quadriprocessor Digital AlphaServer 8200 5/625!

The global solution leads directly to the configuration brass - P1-91 (last column of Table 1). One can notice that the used of the ceramic P1-91 improve significantly all the results, even if it is added with steel. One remarks also, all the global solution are obtained when the length $L$ of the bimorph is maximal and when the thickness of the substrate $h_s$ is minimal according to the given bounds. In fact, it seems to depend only about

Table 1: Optimization of the displacement $\delta$ according to used materials.

| Used materials | steel P1-89 | brass P1-89 | steel P1-91 | **brass P1-91** |
|---|---|---|---|---|
| $h_c$ (mm) | 2 | 1.52 | 2.41 | **1.85** |
| $h_s$ (mm) | 3 | 3 | 3 | **3** |
| $L$ (mm) | 100 | 100 | 100 | **100** |
| $l$ (mm) | 12 | 15.7 | 10 | **13** |
| ceramic volume ($cm^3$) | 2.4 | 2.38 | 2.41 | **2.41** |
| $V$ (V) | 200 | 152 | 241 | **185** |
| $\delta$ ($\mu$m) | 21.3 | 26.7 | 37.5 | **46.5** |

the two following variables: the width $l$ and the thickness of the ceramic $h_c$, but that was not really clear at the beginning of the study, because the ceramic volume is limitate.

Of course, one can extend the model by adding some other materials; nevertheless, on this example, one chooses materials which have good properties for the design of this bimorph, and that doesn't seem necessary to complicate this problem.

# 4 Conclusion

The purpose of this article is to propose a new methodology of conception by solving the inverse problem of the optimal design of this kind of actuators. To generalize this approach some categorical variables must be introduced in the modelization phase. Then, to keep the rigorousness of our design methodology, new deterministic global optimization algorithms were implemented. Our choice were about the extension of interval Branch and Bound techniques by dealing the categorical variables among three principles which permit to keep all the properties of these kinds of method: at the end of the algorithm the global optimum (and also all the solutions) must be enclosed with the given accuracy.

The application of this work on the dimensionning of a piezoelectric actuator: a bimorph, used for the design of a vibrator for magnetometer, was validated and proves that by using this techniques, the global solution can be found quickly and with certainty. Also, our approach could be generalized to the dimensionning of other electromechanical actuators which needs the introduction of categorical variables. That is a first step to solve the inverse problem of the design of piezoelectric or more generally electromechanical actuators.

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