Global Optimization

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Extended Abstract

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In this talk we give an overview of numerical methods for global optimization. In the first part, we describe five optimization problems, the nonconvex mixed-integer nonlinear problem (MINLP), the global optimization problem (GOP), the mixed-integer all-quadratic program (MIQQP), the quadratic binary program (QBP), and the mixed-integer linear program (MILP). For each problem we give some applications.

In the second part, we study reformulations and relaxations of nonconvex optimization problems. In particular, we show how binary constraints can be used to reformulate integrality constraints, if-then-else expressions and piecewise C^2-functions. Moreover we explain how binary constraints can be reformulated by quadratic constraints, and how factorable programs can be reformulated to be separable [TS99].

Next we describe several relaxation techniques. We begin with a theory of Lemárechal and Renaud [LR01] on primal and dual convexification methods. We continue with LP-relaxation of MINLPs [TS99], semidefinite relaxation of MIQQPs [Now02], and convex relaxation of MINLPs.

In the third part, we introduce decomposition methods. We show how splitting schemes can be derived by partitioning the sparsity graph [Now02], and explain Lagrangian decomposition.

The forth part is devoted to solution methods. We describe recent heuristic and exact methods for solving MINLPs. In particular, we introduce some basic concepts of Branch-and-cut algorithms, such as convergency, lower bounds, cuts and global optimality criteria.

In the last part, we present pathfollowing heuristics for solving MINLPs. Such kind of algorithms are particular smoothing heuristics [SSV98] where the problem is smoothed by convex Lagrangian relaxation. We report numerical results for the max-cut problem [AN02], for MINLPs from `MINLPlib` [CG02] and for a MINLP arising in the design of energy conversion systems (`http://www-iam.mathematik.hu-berlin.de/~eopt/`).

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