Review of

L. Jaulin, M. Kieffer, O. Didrit and É. Walter, Applied Interval Analysis, Springer, London 2001.

The book, with the subtitle "With examples in parameter and state estimation, robust control and robotics", presents interval analysis in the context of its applications. While there are a number of good recent books [1, 9, 12, 11, 19] featuring intervals in the context of numerical computations and optimization, they all remain strictly within the limits of mathematics; the only exception is a book by Kolev [11] on interval methods in circuit analysis. Thus the new book is a welcome addition, helping bridge the gap between theory, tools, and applications.

Interval analysis was originally invented by Moore [16, 17] for controlling rounding errors in numerical computations; it serves in this role until today by providing mathematical rigor in computer-assisted proofs in mathematics and mathematical physics. The most conspicuous of these is Hales's proof [8] of Kepler's over 300 years old conjecture that the face-centered cubic lattice is the densest packing of equal spheres in 3-space. Tucker's recent proof [21, 22] that the Lorenz attractor exists also needs interval analysis in a nontrivial way. For other computer-assisted proofs see the recent reviews by Frommer [6] and Feffermann & Seco [5].

A second, initially unexpected, use of interval analysis is based on its ability to provide tools for attacking global questions about nonlinear problems, by allowing a rigorous control of the deviation from nonlinearity. This turns interval techniques into powerful and in some cases indispensable tools in those applications outside mathematics which require the solution of global problems.

The book under review is virtually silent about applications to computer-assisted proofs. It is in the second role – solving global nonlinear problems – that interval analysis is promoted in the book under review, and numerous examples and illustrations show its use in engineering applications.

The book is organized into four parts. Part I gives a short motivation for the book, Part II introduces in Chapters 2–4 operations on intervals, interval vectors (= boxes), and interval matrices, basic properties of various range enclosure forms, two basic workhorses, the branch-and bound principle ('subpavings') and the contracion mapping principle ('contractors'), and enhancements (constraint propagation and linear programming).

This part of the book is not recommended as an introduction to interval analysis per se, where the books mentioned above do a better service. Results are usually given without proofs. Moreover, it seems that the authors worked in isolation from the mainstream of interval analysis, since they introduce numerous new names for traditional concepts ('punctual matrix' for 'point matrix', 'centered inclusion function' for 'centered form', 'convergence rate' for 'convergence order', 'subpaving' for 'partition into boxes', 'Krawczyk contractor' for 'Krawczyk operator', 'external approximation' for 'outer approximation', etc.). Also, a number of concepts are discussed without giving references (no reference to Krawczyk [13], who originated the nonlinear fixed point techniqes; no reference in the context of Taylor arithmetic [4, 10, 14, 20]); the important wrapping effect is mentioned on p.16 with some buzzwords only, but without references where one could get details; see [18] for a recent survey.

Chapter 5 concludes Part II, with discussing variants of a generic algorithm which guarantees to find all solutions (resp. tight inner and outer enclosures of the solution set) of systems of equations and inequalities, global optimization problems, and multilevel optimization problems (including minimax problems as a special case). This is the first time that global solvability of the general multilevel optimization problem has been demonstrated algorithmically.

However, nothing is said about speed, except vague references to the 'curse of dimensionality' (denoting the worst case exponential increase of work with the problem dimension). Indeed, their examples and techniques are currently limited to quite low dimensions. (The authors report on p.5 their unfortunate decision to have different examples run on different hardware and software, so that the reader gets no idea what can be done realistically.) On the other hand, current global optimization codes that combine interval techniques with tools from convex analysis (not treated in the book under review) have been recorded [2, 7] to solve certain nonconvex problems with many hundreds of variables. (An up-to-date survey of global optimization techniques is in [3].)

Part III discusses in three chapters applications to nonlinear parameter estimation, nonlinear state prediction, robust control, and robotics. It is for this part that the book is worth buying and studying.

Chapter 6 poses the parameter estimation problem as an optimization problem; in many cases local optimization leads to poor local minima only, and the best fit requires global optimization. The problem appears in two variants, as least squares problem and as minimax problem, both amenable to the general treatment of Chapter 5. It is interesting that interval techniques not only give point estimates of the parameters, but can capture the set of all parameter combinations that satisfy the desired relations within a given error margin, even when this set has a complicated shape. This is important in certain design problems. Modifications of the general scheme take care of the additional difficulties produced by the presence of outliers and by error-in-variables models. The application to state estimation in discrete dynamical systems requires the use of more intermediate variables, but gives worst case scenarios where the traditional extedned Kalman filter only provides point estimates, which in highly nonlinear situations are of questionable value.

Chapter 7 discusses applications in robust control, allowing the analysis of linear time-invariant dynamical systems depending on uncertain parameters, again in a worst case setting. First, basic notions in control theory are reviewed (without proofs), which reduce the problem to a location analysis of zeros of polynomials or eigenvalues of matrices. The fact that the coefficients of the polynomial or matrix are functions of parameters varying in a box makes the stability verification a difficult, global constraint satisfaction problem or a minimax problem, depending on the precise question asked. These are solved by interval methods using the general methodology.

Chapter 8 contains applications to robotics. In this field, a host of low-dimensional global problems exist whose solvability may have a significant impact on how robots will be built in the future. In particular, so-called parallel robots (MERLET [15]) have a much better load-weight ratio than traditional serial robots, but their highly nonlinear (which means, for engineers, often counterintuitive) nature makes them much more difficult to design and operate. As the chapter shows, interval analysis is ideally suited to solve such problems. Particular examples treated are the configurational analysis of a (parallel) Gough platform, the problem of finding a collision-less path in a known environment, and the problem of self-localization in a partially known environment.

The final part IV deals with implementation issues. Covered are automatic differentiation, IEEE arithmetic and directed rounding, and intervals in $C++$. A compact disc containing a trial version of the new Fortran Forte Compiler of Sun Microsystems (with full interval arithmetic support) comes together with the book.

References

[1] O. Aberth, Precise Numerical Analysis Using C, Academic Press, 1998.

[2] C.S. Adjiman and C.A. Floudas, The αBB Global Optimization Algorithm for Nonconvex Problems: An Overview, Chapter 8 in: From Local to Global Optimization (A. Migdalas et al., eds.) Kluwer, Dordrecht 2001.

[3] C. Bliek, P. Spellucci, L.N. Vicente, A. Neumaier, L. Granvilliers, E. Monfroy, F. Benhamou, E. Huens, P. Van Hentenryck, D. Sam-Haroud and B. Faltings, Algorithms for Solving Nonlinear Constrained and Optimization Problems: The State of the Art. A progress report of the COCONUT project, 2001.
`http://www.mat.univie.ac.at/~neum/glopt/coconut/StArt.html`

[4] J.-P. Eckmann, A. Malaspinas, and S. Oliffson-Kamphorst, A software tool for analysis in function spaces, pp. 147–166 in: Computer Aided Proofs in Analysis, (K. Meyer and D. Schmidt, eds.), Springer, New York 1991.

[5] C.L.Fefferman and L.A.Seco, Interval Arithmetic in Quantum Mechanics, pp. 145–167 in: Applications of Interval Computations (R.B. Kearfott and V. Kreinovich, eds.), Kluwer, Dordrecht 1996.
`http://www-risklab.erin.utoronto.ca/seco/publ.htm`

[6] A. Frommer, Proving conjectures by use of interval arithmetic, pp. 1–13 in: Kulisch, Ulrich; Lohner, Rudolf; Facius Axel: Perspective on Enclosure Methods (U. Kulisch et al., eds.), Springer, Wien 2001.

[7] V. Ghildyal and N.V. Sahinidis, Solving Global Optimization Problems with BARON, Chapter 10 in: From Local to Global Optimization (A. Migdalas et al., eds.) Kluwer, Dordrecht 2001.

[8] T.C. Hales, The Kepler conjecture, Manuscript (1998). math.MG/9811071
`http://www.math.lsa.umich.edu/~hales/countdown/`

[9] E. Hansen, Global Optimization Using Interval Analysis, Dekker, New York 1992.

[10] E. Kaucher, Self-validating computation of ordinary and partial differential equations, pp. 221–254 in: Comuter-Arithmetic (E. Kaucher et al., eds.), Teubner, Stuttgart 1987.

[11] L.V. Kolev, Interval Methods for Circuit Analysis, World Scientific, 1993.

[12] R.B. Kearfott, Rigorous Global Search: Continuous Problems. Kluwer, Dordrecht, 1996.

[13] R. Krawczyk, Newton-Algorithhmen zur Bestimmung von Nullstellen mit Fehlerschranken, Computing 4 (1969), 187–201.

[14] K. Makino and M. Berz, Remainder Differential Algebras and their Applications, pp. 63-74 in: Computational Differentiation: Techniques, Applications, and Tools (M. Berz et al., eds.), SIAM, Philadelphia 1996.

[15] J.-P. Merlet, Parallel Robots, Kluwer, Dordrecht 2000.

[16] R.E. Moore, Interval arithmetic and automatic error analysis in digital computing, Ph.D. thesis, Applied Mathematics and Statistics Laboratories, Report 25, Stanford University, 1962.

[17] R.E. Moore, Interval Analysis, Prentice-Hall, Englewood Cliffs, NJ, 1966.

[18] N.S. Nedialkov and K.R. Jackson, A new perspective on the wrapping effect in interval methods for initial value problems for ordinary differential equations, pp. 219–264 in: Perspectives on Enclosure Methods (U. Kulisch et al., eds.), Springer, Berlin 2001.
`http://www.cs.toronto.edu/NA/reports.html#ned.scan00`

[19] A. Neumaier, Introduction to Numerical Analysis, Cambridge Univ. Press, Cambridge, 2001.

[20] A. Neumaier, Taylor forms – use and limits, to appear.
http://www.mat.univie.ac.at/∼neum/papers.html#taylor

[21] W. Tucker, The Lorenz attractor exists, C.R. Acad. Sci. Paris 328 I (1999), 1197–1202.

[22] W. Tucker, A Rigorous ODE solver and Smale's 14th Problem, Found. Comput. Math. 2 (2002), 53–117.