Classical and quantum field aspects of light

Arnold Neumaier

Fakultät für Mathematik
Universität Wien, Österreich

Lecture at the Institute of Quantum Optics and Quantum
Information of the Austrian Academy of Sciences
Vienna, 29.01.2008

`http://www.mat.univie.ac.at/~neum/papers/physpapers.html#lightslides`

Abstract

This lecture (the first of three) discusses foundational problems on the nature of light revealed by attempts to define a probability concept for photons, quantum models for photons on demand (and their realization through laser-induced emission by a calcium ion in a cavity), models explaining the photo effect, and Bell-type experiments for single photon nonlocality.

This is the attempt of an applied mathematician,

whose main expertise is in the areas of combinatorics, global optimization, and the modeling of uncertainty,

to carry owls to athens,

or photons to a renowned center of quantum optics...

In the past, I repeatedly published work related to quantum mechanics in

- biochemistry (protein folding)

- computing molecular partition functions
  (with V. Mandelshtam)

- computing molecular resonances (with V. Mandelshtam)

- a normal form for general quantum optical networks
  (with U. Leonhardt)

- conceptual foundations of quantum mechanics and
  thermodynamics

I also maintain a continually updated theoretical physics FAQ at

`http://www.mat.univie.ac.at/~neum/physics-faq.txt`

which currently amounts to a 200 page book with 140 sections, mainly on quantum mechanical topics.

The present lecture originates from work towards
a book on **quantum mechanics for mathematicians**.
The intention is

- to provide the material needed for a consistent
  understanding of concepts, methods and use.

- in a mathematically rigorous form,

- without losing sight of the forest for the trees.

So one has to think about how to cast modern developments
into a consistent formal framework.

This has lead me to review the conceptual basis
of quantum optics.

We shall discuss the conceptual issues
in four fairly independent contexts:

1. attempts to define a probability concept for photons,

2. quantum models for photons on demand,

3. models explaining the photo effect, and

4. Bell-type experiments for single photon nonlocality.

1. Photons and probability

Much work in quantum information theory is phrased
in terms of states such as $|1\rangle|2\rangle + |2\rangle|1\rangle$
with very few discrete quantum degrees of freedom.

While very useful for a quick analysis, one loses
some information on the interpretation
of these states as realized in a real system.

The real system happens in space and time
and hence should be described from a
quantum field theory perspective.

Such a description may be of value even in
quantum information engineering when optimizing
a particular system under uncertainty.

According to quantum electrodynamics (QED), the most accurately verified theory in physics, a photon is a single-particle excitation of the free quantum electromagnetic field, more specifically, an eigenstate of the photon number operator with eigenvalue 1.

A single photon has essentially the same degrees of freedom as a classical radiation field.

In a homogeneous medium, where light travels with a constant speed c, its **mode** is characterized by an arbitrary nonzero complex vector potential $\mathbf{A}(\mathbf{x}, t)$ satisfying the free **Maxwell equations**, which in the radiation gauge take the form

$$c^{-2}\partial_t^2 \mathbf{A}(\mathbf{x}, t) = \Delta \mathbf{A}(\mathbf{x}, t), \quad \nabla \cdot \mathbf{A}(\mathbf{x}, t) = 0,$$

expressing the zero mass and the transversality of photons.

Modes $\mathbf{A}$ satisfying these equations and differing by a space-independent phase factor describe the same photon state $|\mathbf{A}\rangle$.

Thus the Hilbert space of **photon wave functions** is the space of divergence-free vector fields in the spatial region Ω of interest, corresponding to the possible photon modes restricted to a fixed time.

For $\Omega = \mathbb{R}^3$, the correct inner product (which leads to a spin 1 representation of the Poincaré group or conformal group) is given by

$$\langle \mathbf{A} | \mathbf{A}' \rangle_t = \frac{1}{2\pi^2 \hbar c} \int \frac{d\mathbf{x} d\mathbf{y}}{|\mathbf{x} - \mathbf{y}|^2} (\nabla \times \mathbf{A}(\mathbf{x}, t))^* (\nabla \times \mathbf{A}'(\mathbf{y}, t)).$$

For more general Ω, there are additional boundary terms, and the normalization constant changes.

To be able to talk consistently about the probability
of a photon being in a partickular subregion of Ω,
one needs a family of commuting projection operators
which restricts a mode to such a subregion.

The projection operators are defined as multiplication
with the corresponding characteristic function.

But the fact that photons are transverse has the consequence
that multiplying a photon mode $\mathbf{A}(\mathbf{x}, t)$ by a space-dependent
scalar function does not result in another photon mode.

Thus the projection operators lead out of the Hilbert space.

This implies that **there is no consistent notion of
probability density for photons**.

The textbook interpretation of Schrödinger wavefunctions
is not applicable to photons.

An even stronger version of expressing this is that
it is impossible to find for photons a vector-valued
position operator with commuting coordinates
which transforms under rotations like a vector.

This impossibility has been observed without proof
by NEWTON & WIGNER in 1949, and was proved
in 1962 by WIGHTMAN.

An excellent review about failed attempts to circumvent
this conclusion was recently given by BIALYNICKI-BIRULA
in `http://lanl.arxiv.org/abs/quant-ph/0508202`

Note that different gauge fixing recipes cannot
alter our conclusion since the same spin 1 representation
of the Poincaré group must result.

Thus all the resulting descriptions are isomorphic.

Thus it is impossible to talk consistently about the probability for a photon being located in a particular subregion of Ω.

This rules out the interpretation of photons as point particles.

On the other hand, photons prepared in a laboratory are located there.

Thus there must exist a weaker form of localization.

This is consistent with the lack of a well-defined probability distribution if we assume that **Photons are intrinsically nonlocal objects**.

Compare this to extended objects like cars.
The probability of a car being in a garage is also not completely well-defined; there is a counting ambiguity when a car entered the garage only half way.

However, while there is no natural probability density,
the expression analogous to the probability density
of a Schrödinger particle,

$$\rho(\mathbf{x}, t) := |\mathbf{A}(\mathbf{x}, t)|^2,$$

integrates (if $\varepsilon_0 = \mu_0 = 1$) over space to the
total mean energy of the photon at time t.

Hence $\rho(\mathbf{x}, t)$ can be regarded as the
mean energy density of the photon.

2. Photons on demand

To get close to the modeling of experiments, we consider quantum models for photons on demand and their realization through laser-induced emission by a **single** calcium ion in a cavity.

The exposition is based on work at the
Max Planck Institute of Quantum Optics in Garching (Germany)
in the laser physics group of the late Professor Walther.

For details, see the paper

M Keller, B Lange, K Hayasaka, W Lange and H Walther,
A calcium ion in a cavity as a controlled single-photon source,
New Journal of Physics 6 (2004), 95.

from which some figures are taken, and which discusses
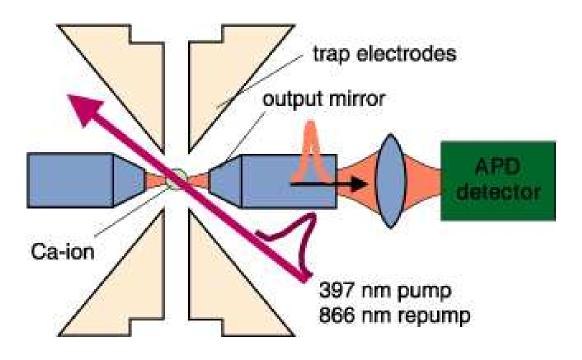an explicit model of their experimental setting.

Figure 16. Experimental set-up for the generation of single-photon pulses with an ion-cavity system. The drawing shows a cross-section through the trap, perpendicular to the trap axis.
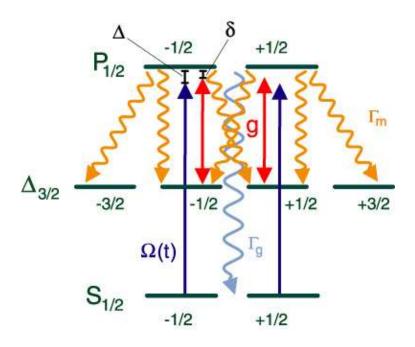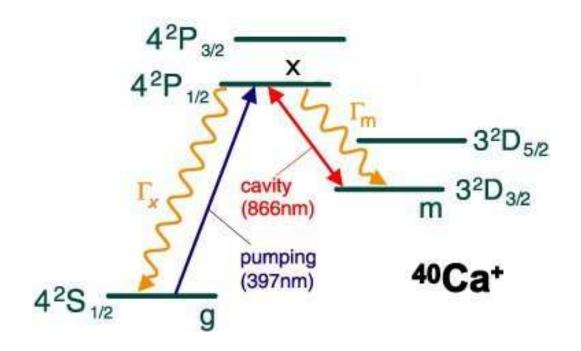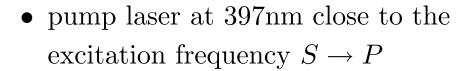
Figure 5. Scheme of the eight-level model on which we base our numerical calculations. Pump and cavity field are assumed to be linearly polarized in the direction of the quantization axis. For clarity, the four possible spontaneous decay transitions to the ground state are represented by a single arrow.
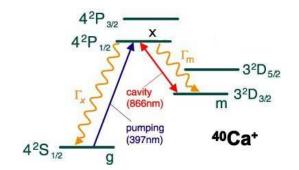
In their paper, KELLER et al. discuss in detail
a model based on the following simplified level scheme
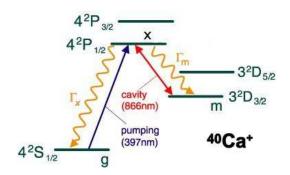which ignores the fine structure of the Ca^+ states.

- A **single** ion is localized in the cavity for many hours

- pulsating external fields (lasers) with a total cycle time 100kHz give a predictable rate of single photons

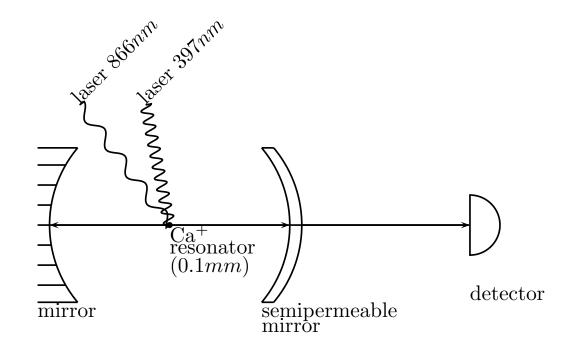- pump laser at 397nm close to the excitation frequency $S \to P$

- Repeated excitation to P and decay to S until decay into the metastable D state; then inactive

- $\Rightarrow$ produces **exactly** one photon (not counting losses)

- reexcite ion into excited state with a reset laser at 866nm, until it falls back into the ground state
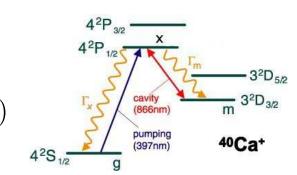
- ground state g, metastable state m, excited state x of Ca^+

- photons $\gamma_{\text{cavity}}, \gamma_{\text{pump}}, \gamma_{\text{reset}}$

- electron e bound in detector

Active processes

- a: $\gamma_{\text{cavity}} \rightleftharpoons \gamma_{\text{cavity}}$ (cavity detuning)

- b: $g + \gamma_{\text{pump}} \rightleftharpoons x$ (excitation)

- c: $x \rightleftharpoons m + \gamma_{\text{cavity}}$ (decay to metastable state)

- d: $\gamma_{\text{cavity}} + e \rightleftharpoons \emptyset$ (photodetection)

- e: $m + \gamma_{\text{reset}} \rightleftharpoons x$ (ion reset)
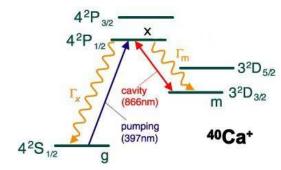
Only a, b, c are modelled explicitly by KELLER et al..

But d, e can be modelled similarly.

Interaction picture model

by KELLER et al.

(without reset and photodetection)

- a = annihilator of cavity mode of photon

- $b = |g\rangle\langle x|$

- $c = |m\rangle\langle x|$

- Hamiltonian
  $$H = \hbar(\delta a^* a + \Delta |g\rangle\langle g| + 2\,\mathrm{Re}\,(\Omega(t)b^* + \mu a c^*))$$

- $\delta = \omega_{\text{cavity}} - \omega_{xm}$ cavity detuning

- $\Delta = \omega_{\text{pump}} - \omega_{gx}$ pump detuning

- $\Omega(t)$ classical pulse shape of pump laser

- μ (KELLER's g) ion–cavity coupling strength
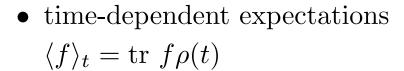
- $\kappa = 0.02\,\Gamma_x$ cavity loss rate

- $\Gamma_x \approx 138$ MHz (KELLER's Γ_g)
  spontaneous decay rate into ground state $4\,^2S_{1/2}$

- $\Gamma_m \approx 11$ MHz spontaneous decay rate into metastable state

To account for losses, the dynamics of the density matrix
is set up in the form of a

Lindblad master equation

$$
\begin{aligned}
\dot{\rho} \quad = -\tfrac{i}{\hbar}[H, \rho] \quad &+ \kappa(2a\rho a^* - a^*a\rho - \rho a^*a) \\
&+ \tfrac{\Gamma_x}{2}(2b\rho b^* - b^*b\rho - \rho b^*b) \\
&+ \tfrac{\Gamma_m}{2}(2c\rho c^* - c^*c\rho - \rho c^*c)
\end{aligned}
$$

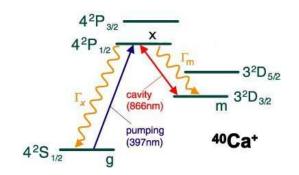Note that the master equation is an equation for transition rates; probabilities are obtained by integration over time.

- time-dependent expectations
  $\langle f \rangle_t = \operatorname{tr} f \rho(t)$

- time dependent emission rate
  $p(t) = 2\kappa_{\mathrm{tr}} \langle a^* a \rangle_t$
  ($2\kappa_{\mathrm{tr}}$ intensity transmission rate)

- probability of photon emerging from the cavity
  $\eta_{\mathrm{photon}} = \int_0^\infty p(t) dt$

- single-photon efficiency
  $\eta_{\mathrm{abs}} = (\kappa/\kappa_{\mathrm{tr}} - 1)\eta_{\mathrm{photon}}$

23

The Hilbert space on which the master equation is based is the tensor product of a single mode Fock space for the cavity photon and a 3-mode space for the Ca^+ ion.

An orthonormal basis of the space is given by the kets $|n, k\rangle$, where $n = 0, 1, \dots$ is the photon occupation number and $k \in \{g, x, m\}$ labels the ion level.

The structure of the Hamiltonian and the dissipation terms in the master equation is such that if the system is started in the ground state $|0, g\rangle$, it evolves to a mixed state in which the photon number is never larger than 1.

Thus multiphoton states do not contribute at all, and one can truncate the cavity photon Fock space to the two modes with occupation number $n = 0, 1$, without changing the essence of the model.

Of interest for the photon production is the projection
of the density matrix to the photon space, obtained
by tracing over the ion degrees of freedom. This results
in an effective time-dependent photon density matrix

$$\rho_{\text{photon}}(t) = \begin{pmatrix} \rho_{00}(t) & \rho_{01}(t) \\ \rho_{10}(t) & \rho_{11}(t) \end{pmatrix},$$

where
$\rho_{11}(t) = p(t)$ is the probability density of finding a photon,
$\rho_{00}(t)$ is the probability density of finding no photon, and
$\rho_{01}(t) = \rho_{10}(t)^*$ measures the amount of entanglement
between the 1-photon state and the vacuum state.

Semidefiniteness of the state requires $|\rho_{01}| \leq \sqrt{p(1-p)}$.

Assuming for simplicity that we have approximate equality,
ρ_{photon} is essentially rank one,
$$\rho_{\text{photon}}(t) \approx \psi(t)\psi(t)^*, \quad \psi(t) = s(t)|0\rangle + c(t)|1\rangle,$$
where $s(t)$ and $c(t)$ are functions with $|s(t)|^2 + |c(t)|^2 = 1$,
determined only up to a time-dependent phase factor.
In particular, we may take $c(t)$ to be real and nonnegative.

Thus, in the approximation considered,
the quantum electromagnetic field is in a superposition
of the vacuum mode and the single-photon field mode,
with a 1-photon amplitude $c(t) = \sqrt{p(t)}$
that varies with time and encodes the
probability density $p(t)$ of detecting a photon particle.

In the actual experiments, $p(t)$ has a bell-shaped form,
and the total photon detection probability,
referred to as the **efficiency**, is significant,
but smaller than 1.

Discarding the vacuum contribution corresponding
to the dark, unexcited cavity, and giving up
the interaction picture by inserting the field description
$|1\rangle_t = e^{-i\omega t}\psi_0(\mathbf{x})$ of the photon mode,
the (now time-dependent) 1-photon state takes the form
$$\mathbf{A}_{1\mathrm{photon}}(\mathbf{x}, t) = \sqrt{p(t)}e^{-i\omega t}\psi_0(\mathbf{x}).$$

At this stage one notices a minor discrepancy
with the field description, since the 1-photon state is
no longer an exact solution of the Maxwell equations.

To correct this deviation from Maxwell's equations, one has to work with quasi-monochromatic modes and the paraxial approximation.

This should result in a more accurate time-dependent 1-photon state, describing a bell shaped electromagnetic field pulse $\mathbf{A}(x, t)$ which solves the Maxwell equation exactly.

A detailed model for this is work in progress.

We now add the reset mechanism to get a
continuous pulsed photon stream.

Thus we consider a periodic sequence
of excitation-reset cycles of the ion in the cavity.

As before, we find that the electromagnetic field corresponding
to the sequence of pulses is a single, periodically excited
1-photon mode of the electromagnetic field.

Thus what appears at the photodetector as
a **sequence** of photon particles arriving
is from the perspective of quantum electrodynamics
the manifestation of a **single** nonstationary,
pulsed 1-photon state of the electromagnetic field!

We now discuss the consequences of this
somewhat surprising analysis.

What is a photon?

In 2003, a special issue of the
Optics & Photonic News was devoted to this topic.

5 articles by 8 experts – 5 different answers.
Thus this is still a debated question.
This justifies the present attempt of giving a new answer.

Our analysis of the photon on demand model concluded
that a **fixed**, periodically excited 1-photon field state
carries a **sequence** of controllable and measurable
energy bursts – **photons on demand**.

Thus the **QED photon state** concept
(a singly excited quantum state of the free electromagnetic field)
differs from the **photon particle** concept
(addressing moving lumps of energy).

In view of the fact that

- already classically, a point particle picture of light
  is valid only to the extent to which the
  geometric optics approximation is valid,

- there is no consistent photon probability
  but a consistent energy density,

- the QED photon mode picture differs from
  the photo detection particle picture,

I want to suggest the following

Definition.
A photon particle is a local concentration of energy in a photon
field mode, integrating to a total energy of $h\nu = \hbar\omega$.

This seems to be in good agreement with both QED and the current experimental practice.

The spin 1 representation of the Poincare group determines the 1-photon Hilbert space.

This Hilbert space consists of all possible 1-photon field modes.

A photon particle is a particular field mode whose energy is approximately spatially localized.

Thus the **QED photon** is a global state of the whole space, a time-dependent solution of the Maxwell equation.

It acts as a carrier of **photon particles**, which are extended but localized lumps of energy moving with the speed of light along the beam defined by a QED photon state.

It is interesting to note that Colosi & Rovelli 2006 arrived
at a similar conclusion from a completely different perspective.

They argue from quantum general relativity,
starting with the Unruh effect.

The Unruh effect demonstrates that what appears as
a vacuum field state to an observer at rest
appears to an accelerated observer as a multiphoton field state.

That being a particle depends on the frame of reference
seems quite odd.

Some authors (Davies 1984, Zeh 1993, 2003) even tried
to use the Unruh effect to deny the existence of particles in
a fundamental quantum field theory.

Questions

The above analysis suggests to consider correlation experiments
between **preparation** and **measurement** of photons
rather than between different measurements of entangled photons.

i.e., between the exciting laser source and the photodetection
when a sequence of low efficiency lumps of energy are sent.

In particular, what happens to the time correlations
when (as in actual realizations of photons on demand)
the lumps are significantly smaller than
one full quantum of energy $\hbar\omega$;
i.e., what are the observable consequences of
"fractions of photon particles"?

Are there already such experiments?

As laser pulses get closer, photon particles are more and more difficult to separate and the time-dependent mode becomes effectively continuously modulated.

What is the discrete response (experimental vs. predicted) to continuously varying laser settings?

One possibility is that an integrated measure of the energy transported by the beam, discounted for losses in the detector, could be the factor determining the stochastic process at the detector.

It would be interesting to check this experimentally and confirm this, or replace it by a more accurate picture of what really happens.

A time-dependent treatment of the detection response would be needed to get theoretical predictions.

3. Photodetection

Although a quantum effect, the photo effect
has nothing to do with quantum properties of light.

A frequently heard opinion, popularized by GLAUBER,
"photons are what photodetectors count",
is correct only if one also admits **classical photons**.

In their recent survey "The concept of the photon",
MUTHUKRISHNAN et al. write: "In fact, it can be shown
that the essence of **the photoelectric effect does not
require the quantization of the radiation field,**
a misconception perpetuated by the mills of textbooks".

Some people, such as the Nobel prize winner
Willis LAMB 1995 even take this as an indicator
that *"there is no such thing as a photon"*.

That photodetection works correctly already with
(deterministic) classical fields and (noisy) quantum detectors
applies only to normal light or laser light,
for which most quantum effects are reproducible
with classical electromagnetic fields (STROUD & JAYNES).

Of course, quantum field theory is needed
to explain the nonclassical correlations observed
when experimenting with nonclassical light.

See, e.g., CLAUSER 1974 for the limitations of
classical fields in case of coincidence measurements.

So let us consider the case of normal light.

Immediately after Heisenberg and Schrödinger
discovered the modern form of quantum mechanics,
two distinct quantum treatments of the photo effect
were given in 1927 by DIRAC and WENTZEL.

DIRAC used "light quanta", now called photons,
after LEWIS 1926. (But LEWIS' photons were
supposed to be conserved, while real photons are not.)

WENTZEL's derivation was instead based on
the response of a **quantum detector**
to a **classical** electromagnetic field.

A first observation is that both the quantum and
the classical treatment produce detection rates
(rather than a **single** detection probability)
as the response to a monochromatic 1-photon mode
of the electromagnetic field
(clearly a nice QED **single**-photon state).

This fits the proposed distinction between a photon field state
and the many photon particles it can carry.

But how can a classical treatment be
reconciled with a photon particle picture?

As standard semiclassical treatment
(with a classical treatment of light
and a quantum treatment of electrons)
we may consider the treatment in the
quantum optics book by MANDEL & WOLF 1995.

A rigorous treatment of the photo effect for classical light
requires the solution of the Dirac equation for an electron
in a periodic potential representing a crystal,
with a monochromatic external electromagnetic field.

Fearn & Lamb (1991) treat the bare bones of the photoelectric effect with classical light in a simpler, exactly solvable toy approximation.

The electrons are reduced to a 1-dimensional quantum system in a δ-function potential, and light is treated as a time-dependent external field contribution to the Hamiltonian.

The electron in these and other models shows a response correctly reproducing the behavior of a single photodetector fed with normal light.

This shows conclusively that **not photons but electrons**, the only quantum mechanical system in these models, are responsible for the quantum effects in the photodetector.

The **probabilistic results** of low intensity photodetection
are therefore the consequence of the interaction with
the ensemble of highly localized particles in the
macroscopic detector: **myriads of bistable electrons**,
one or more in each minimum of the
periodic potential of the crystal.

They are **not** the result of strangely and randomly
behaving quantum particles called photons,
allegedly transported by the beams of light
and materializing as a click in the detector.

Instead, quantization is the inevitable response of
a collection of independent bistable detection elements
to a continuous signal – whether the signal is a classical
or a quantum signal. Probably it does not even matter
whether the detection elements are classical or quantum!

Thus we are forced to the conclusion that
there are no random photons
in a monochromatic beam.

This **lack of inherent randomness**
is also the ultimate reason for
why it is possible to produce
deterministically **photons on demand**.

On the other hand, the intuitive appeal of photons as particles, and time and coincidence measurements of photodetection events, require an explanation.

The suggested distinction between photon field modes (in the sense of QED) and photon particles (as lumps or "wave packets" of radiation energy) **reconciles the intuition with the formal requirements**. But some photons (such as those in plane waves) are very delocalized and only artificially separable into quanta of energy $h\nu = \hbar\omega$.

Since the definition of a photon as a localized concentration of energy in a field mode is independent of quantum mechanics, **there are also classical (and deterministic) photon particles,** thus giving the semiclassical models of the photo effect a natural interpretation.

Questions

Can one produce very low intensity monochromatic
(i.e., stationary rather than pulsed) modes of light?

What about predetermined low intensity,
continuously modulated quasi-monochromatic modes?

What is their behavior under photodetection?

One expects some sort of modulated Poisson process. . . .

4. Single photon nonlocality

Papers discussing experiments involving the entanglement of single photons and associated Bell-type experiments include
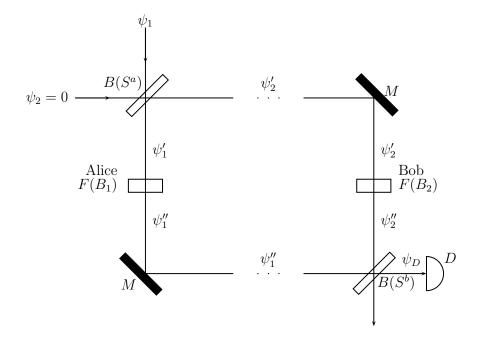
TAN et al. 1991, HARDY 1994, PERES 1995,
GERRY 1996, SPREEUW 1998, LEE & KIM 2000,
BJÖRK et al. 2001, BEIGE et al. 2002, IKRAM & SAIF 2002,
KIM 2003, BABICHEV et al. 2004, HESSMO et al. 2004,
CAN et al. 2005, VAN ENK 2005, BARTLETT et al. 2006,
WILDFEUER et al. 2006, DUNNINGHAM & VEDRAL 2007.

In particular, HESSMO et al. and WILDFEUER et al. verify experimentally the prediction of single-particle nonlocality by TAN et al., and BABICHEV et al. discuss the detection loophole for single-particle Bell inequality violation.
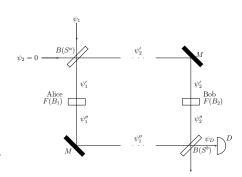
I want to propose a new experiment of Bell type (NEUMAIER 2007).

Consider a fixed, symmetric arrangement of optical instruments
in the form of a long-distance Mach-Zehnder interferometer
with intermediate optical filters under the control of
Alice (here) and Bob (far away).

Alice prepares the source; Bob performs the measurements.

The paths have exactly the same length.

Each $B(S)$ is a non-polarizing beam splitter
with a fixed unitary scattering matrix $S \in \mathbb{C}^{2\times 2}$.

Each M is a mirror.

Each $F(B)$ is a linear, not depolarizing filter transforming
the unnormalized wave function ψ into $B\psi$,
with an adjustable 2×2 transformation matrix B.

D is a detector registering an incident photon
with probability q.

The setting entangles the momentum degree of freedoms
(beam direction) with the polarization degrees of freedom.

Hence a quantum effect is expected to occur.
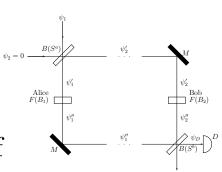
Analysis with hidden variables

Here we make the following assumptions:

(i) The source of beam 1 produces an ensemble of photons which is in the classical, submicroscopic state λ with probability density $p(\lambda)$.

(ii) At a beam splitter, each photon moves along one of the two possible outgoing beams.

(iii) Whether a photon created at the source in state λ reaches the detector after passing the kth filter depends only on B_k and λ. (This is reasonable since one can make a beam completely dark, in which case it carries no photons.)

(iv) The conditional probability of detecting a photon which is in state λ and passes through filter k when $B_k = B$ and $B_{3-k} = 0$ is given by a functional expression $p_k(B, \lambda)$.

These assumptions are **identical** to those made in the proof of Bell inequalities.

In particular, the assumptions include the possibility that the instruments treat photons in different classical states λ differently.

Indeed, (ii) and (iii) hold with

$$p_k(B, \lambda) = p_{k1}^a(\lambda) \cdot P(B, \lambda) \cdot p_{1k}^b(\lambda) \cdot p(\lambda)$$

if the following three conditions hold:

(iiia) A photon in state λ entering the beam splitter $s \in \{a, b\}$ in beam k leaves it in beam l with probability $p_{lk}^s(\lambda)$.

(iiib) A photon in state λ entering the filter $F(B)$ leaves it with probability $p(B, \lambda)$.

(iiic) A photon in state λ reaching the detector is detected with probability $p(\lambda)$.
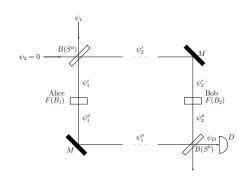
Analysis with quantum mechanics

Here we use unnormalized wave functions $\psi \in \mathbb{C}^2$
to denote the state of an ensemble of photons in
a quasi-monochromatic beam of completely polarized light.

ψ is scaled such that $|\psi|^2 = \psi^* \psi$ equals the probability
of presence of a photon in the beam in a fixed
time interval $[t_0, t_1]$; in particular,
dark beams without any photons are described by $\psi = 0$.

Optical filters consisting of a combination of polarizers
are described by linear transformations of ψ of rank one.

Arbitrary such linear transformations $\psi \to B\psi$ $(B = uv^*)$
can be realized as long as $|u||v|$ is significantly smaller than 1.

The sources of the two beams are assumed
to produce completely polarized light
described by the unnormalized
wave functions ψ_k $(k = 1, 2)$.

The wave functions are scaled such that initially
$\psi_1^* \psi_1 + \psi_2^* \psi_2 = 1$,
corresponding to the presence of just one photon
in both beams together.

(In the actual experiment proposed, $\psi_2 = 0$,
hence the photon is initially in beam 1.)

The scattering matrices of the two beam splitters are fixed in the experiment and given by
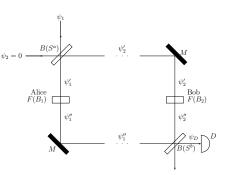

$$S^j = \begin{pmatrix} t_1^j & r_2^j \\ r_1^j & t_2^j \end{pmatrix} \quad (j = a, b),$$

where t_k^j are the transmission coefficients
and r_k^j the reflection coefficients of the two beams.

Two input beams of the beam splitter
with wave functions ψ_1 and ψ_2
are transformed into

$$\begin{pmatrix} \psi_1' \\ \psi_2' \end{pmatrix} = S^j \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} t_1^j \psi_1 + r_2^j \psi_2 \\ r_1^j \psi_1 + t_2^j \psi_2 \end{pmatrix}.$$

We denote by $p(B_1, B_2)$ the probability of detection of a photon in the given arrangement, where B_1 and B_2 are the transformation matrices for the filters in beam 1 and beam 2, respectively, and the second beam is initially dark.
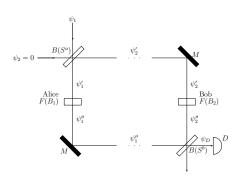

By recording enough photons under various settings of B_1 and B_2, we can determine the difference

$$\Delta(B_1, B_2) := p(B_1, B_2) - p(B_1, 0) - p(0, B_2),$$

in principle to arbitrary accuracy.

We determine the expected dependence
of $\Delta(B_1, B_2)$ on the arguments
in two ways, first assuming a
classical hidden variable model,
and then assuming quantum mechanics.

Hidden variable prediction:

$$\Delta(B_1, B_2) = 0.$$

Quantum prediction ($\rho_1 = \psi_1 \psi_1^*$):

$$\Delta(B_1, B_2) = 2q|t_1^a t_1^b r_1^a r_2^b|^2 \,\mathrm{Re}\,\mathrm{tr}\,\rho_1 B_1^* B_2.$$

This permits an experimental check on which assumption gives an
adequate description of the situation.

Of course, one expects the quantum prediction to be correct.

The nonlinearity in the squared amplitude formula
for the probability is responsible for a nontrivial
interference term in the quantum case.

Thus, comparable to destructive interference in
two-slit experiments, constructive interference is the source
for the discrepancy between the two predictions.

Since the experiment does not involve photon
correlation measurements, the quantum analysis even holds
for mixed states and multiphoton input, provided that
one takes ρ as the effective single-photon density matrix
of the multiphoton state, normalized such that tr ρ equals
the mean number of photons in the fixed time interval $[t_0, t_1]$.

This allows the experiment to be carried out with strong laser light.
In this case, the number of photons is enormous, and the
probabilities turn into essentially deterministic current strengths.

Since the quantum mechanics of a single photon is
that of the Maxwell equations, the experiment
can be explained by the **classical** Maxwell equations,
upon interpreting the photon number detection rate
as being proportional to the beam intensity.

This is a classical description, not by classical particles
(photons) but by classical waves.

Thus a classical wave model for quantum mechanics
is **not ruled out** by experiments demonstrating
the violation of the traditional hidden variable
assumptions.

Therefore the traditional hidden variable assumption only amounts to a **hidden classical particle assumption**.

And the experiments demonstrating their violation only disprove classical models with particle structure.

All proofs of Bell type results (including the present argument) **become invalid** when "particles" have a temporal and spatial extension over the whole experimental domain, with an internal structure that is modified when interacting in a beam splitter.

We conclude that

classical field theory models for a quantum phenomenon
are not excluded by traditional no-go theorems
for hidden variables.

A quantum description of light is needed
not for entanglement per se
(which also exists in classical field theories)

but only to accurately model correlations of
multiphoton entanglement (where nobody
doubts the need for quantum mechanics).

Summary

We reviewed the nonexistence of a probability concept for photons, the existence of a well-defined energy density, quantum models for photons on demand, semiclassical models explaining the photo effect, and Bell-type experiments for single photon nonlocality.

All this suggests that there are two distinct notions of photons, namely **photon field states** describing excitiations of the free electromagnetic field and **photon particles** which are localized wave packets of almost sharp frequency with a definite energy contents.

Moreover, there are classical interpretations for the photo effect and for single photon nonlocality, based on the Maxwell equations and photons as extended wave packets – rather than a classical point particle picture, which is excluded by Bell-type arguments.

So, how classical is quantum mechanics?

The story is going on...

But for this lecture, it is a good time to stop.

The second part of the story can be found at

`http://www.mat.univie.ac.at/~neum/papers/physpapers.html#optslides`

References are given below.

I hope to have shown
you the familiar owls
from a quite unusual,
interesting perspective.

Thank you for
your attention!

Arnold Neumaier

http://www.mat.univie.ac.at/~neum

References

[1] M. D'Angelo, A. Zavatta, V. Parigi, and M. Bellini1, Tomographic test of Bell's inequality for a time-delocalized single photon, Phys. Rev. A 74, 052114 (2006).

[2] S.A. Babichev, J. Appel and A.I. Lvovsky, Homodyne Tomography Characterization and Nonlocality of a Dual-Mode Optical Qubit, Phys. Rev. Lett. 92 (2004), 193601.

[3] I. Bialynicki-Birula, Photon wave function, Manuscript (2005). http://arxiv.org/abs/quant-ph/0508202

[4] J. F. Clauser, Experimental distinction between the quantum and classical field-theoretic predictions for the photoelectric effect, Phys. Rev. D 9 (1974), 853–860.

[5] D. Colosi and C. Rovelli, Global particles, local particles gr-qc/0409054

[6] A.H. Compton, A Quantum Theory of the Scattering of X-rays by Light Elements, Phys. Rev. 21 (1923), 483–502.

[7] P.C.W. Davies, Particles do not exist, p. 66–77 in: Quantum Theory of Gravity (S. Christensen, ed.), Bristol 1984.

[8] P.A.M. Dirac, The quantum theory of the emission and absorption of radiation, Proc. Roy. Soc. London A 114 (1927), 243–245.

[9] H. Fearn and W.E. Lamb, Jr., Corrections to the golden rule, Phys. Rev. A 43 (1991), 2124–2128.

[10] B. Hessmo, P. Usachev, H. Heydari and G. Björk, Experimental demonstration of single photon nonlocality, Phys. Rev. Lett. 92 (2004), 180401.

[11] M. Keller, B Lange, K Hayasaka, W Lange and H Walther, A calcium ion in a cavity as a controlled single-photon source, New Journal of Physics 6 (2004), 95.

[12] W.E Lamb, Jr., Anti-Photon, Applied Physics B 60 (1995), 77–84. Reprinted in: W.E Lamb, Jr., The interpretation of quantum mechanics, Rinton Press, Princeton 2001.

[13] U. Leonhardt, Measuring the Quantum State of Light, Cambridge, 1997.

[14] U. Leonhardt, Quantum Physics of Simple Optical Instruments, Rep. Prog. Phys. 66 (2003) 1207–1250. quant-ph/0305007

[15] G.N. Lewis, The conservation of photons, Nature 118 (1926), 874–875.

[16] L. Mandel and E. Wolf, Optical Coherence and Quantum Optics, Cambridge University Press, 1995.

[17] A. Muthukrishnan, M.O. Scully and M.S. Zubairy, The concept of the photon – revisited, pp. 18–27 in [21].

[18] A. Neumaier, A theoretical physics FAQ. Manuscript
(2004–2008, continuously updated).
`http://www.mat.univie.ac.at/~neum/physics-faq.txt`

[19] A. Neumaier, A simple hidden variable experiment,
Mansuscript (2007). Arxiv:0706.0155
This paper contains all quoted references
on single photon entanglement.

[20] T.D. Newton and E.P. Wigner, Localized states for elementary
systems, Rev. Mod. Phys. 21 (1949), 400–406.

[21] C. Roychoudhuri and R. Roy (eds.), The Nature of Light:
What Is a Photon? Optics and Photonics News, October 2003
`http://www.osa-opn.org/abstract.cfm?URI=OPN-14-10-49`

[22] C.R. Stroud, Jr. and E.T. Jaynes, Long-Term Solutions in
Semiclassical Radiation Theory, Phys. Rev. A 1 (1970),
106–121.

[23] G. Wentzel, Über die Richtungsverteilung der Photoelektronen, Z. Phys A 41 (1927), 828–832.

[24] J.A. Wheeler and W. H. Zurek, Quantum theory and measurement. Princeton Univ. Press, Princeton 1983.)

[25] A. S. Wightman, On the Localizability of Quantum Mechanical Systems, Rev. Mod. Phys. 34 (1962), 845–872.

[26] C.F. Wildfeuer, A.P. Lund and J.P. Dowling, Maximally Path-Entangled Number States Violate a Bell's Inequality, Manuscript (2006). arXiv:quant-ph/0610180v3

[27] C.F. Wildfeuer, A.P. Lund and J.P. Dowling, Strong violations of Bell-type inequalities for path-entangled number state, Phys. Rev. A 76, 052101 (2007)

[28] H.D. Zeh, There are no quantum jumps, nor are there particles! Phys. Lett. A 172 (1993), 189–192.

[29] H.D. Zeh, There is no "first" quantization, Phys. Lett. A 309 (2003), 329–334.