Optical models for quantum mechanics

Arnold Neumaier

Fakultät für Mathematik
Universität Wien, Österreich

Lecture given on February 16, 2010 at the

Institut für Theoretische Physik

Universität Giessen, Germany

http://www.mat.univie.ac.at/~neum/papers/physpapers.html#optslides

Abstract

This lecture (the second of three) discusses work
uncovering classical aspects of quantum mechanics.

It is based on an analysis of polarized light,
of the meaning of quantum ensembles in a field theory,
of classical simulations of quantum computing algorithms,
and resulting optical models for the simulation
of quantum mechanics.

In particular, it is shown that
classical second-order stochastic optics
is precisely the quantum mechanics of a single photon,
with all its phenomenological bells and whistles.

In the previous lecture

Classical and quantum field aspects of light

http://www.mat.univie.ac.at/~neum/papers/physpapers.html#lightslides

(whose content is assumed for this lecture), I reviewed
the nonexistence of a probability concept for photons,
the existence of a well-defined energy density,
quantum models for photons on demands,
semiclassical models explaining the photo effect,
and Bell-type experiments for single photon nonlocality.

All this suggests that there are two distinct notions of photons, namely **photon field states** describing excitations of the free electromagnetic field and **photon particles** which are localized wave packets of almost sharp frequency with a definite energy content.

A plane electromagnetic wave and a pulsed laser beam may both represent a photon (considered as a photon field state), while only the second may be viewed as a sequence of photon particles.

Moreover, there are classical interpretations for the photo effect and for single photon nonlocality, based on the Maxwell equations (rather than a point particle picture, which is excluded by Bell-type arguments).

In this lecture, we continue the analysis,
working towards a more classical view of quantum mechanics.

Themes contributing to the final conclusions:

1. Polarized light
2. What exists in a beam of light?
3. The position of quantum objects
4. Making sense of fuzzy concepts
5. Probability via expectation
6. What is an ensemble of fields?
7. Classical quantum computing
8. Simulating quantum mechanics

1. Polarized light

Polarized light is the simplest quantum phenomenon.

It is the only quantum phenomenon
that was understood quantitatively already
before the birth of quantum mechanics in 1900.

Polarized light was discovered by Christiaan Huygens in 1690.

The transformation behavior of rays of completely polarized light
was first described by Etienne-Louis Malus 1809
(who coined the name "polarization"),
and that of partially polarized light by George Stokes 1852.

In modern terminology, the behavior is identical to that
of a qubit in a pure resp. mixed state.

The transverse nature of polarization was discovered
by Augustin Fresnel 1866, and the description in terms of
(what is now called) the Bloch sphere by Henri Poincare 1892.

In modern terminology, polarization is a manifestation
of the massless spin 1 nature of the unitary representation
of the Poincare group defining photons.

Polarization is a macroscopic phenomenon.
The counterintuitive features of quantum mechanics
irritating the untrained intuition are still absent.
Experiments with polarization filters are easy to perform;
probably they are already known from school.

Thus polarization experiments show the basic principles
of quantum mechanics in a clean and transparent way.

But polarization was recognized as a quantum phenomenon
only much later, when quantum mechanics was
already fully developped.

Norbert Wiener 1930 exhibited a description
in terms of the Pauli matrices and wrote:

"It is the conviction of the author that this analogy"
between classical optics and quantum mechanics
"is not merely an accident, but is due to a
deep-lying connection between the two theories".

Polarization in a nutshell.

A ray of polarized light of fixed frequency is characterized by a state, described equivalently by a real **Stokes vector**

$$S = (S_0, S_1, S_2, S_3)$$

with

$$S_0 \geq |\mathbf{S}| = \sqrt{S_1^2 + S_2^2 + S_3^2},$$

or by a **coherence matrix** (or density matrix), a complex positive semidefinite 2×2 matrix ρ.

These are related by $\rho = \frac{1}{2}(S_0 - \mathbf{S} \cdot \sigma)$, where σ is the vector of Pauli matrices.

$\mathrm{tr}\ \rho = S_0$ is the intensity of the beam.

$p = |\mathbf{S}|/S_0$ is the degree of polarization.

A linear, non-mixing (not depolarizing) instrument is characterized by a complex 2×2 **Jones matrix** T.

The instrument transforms an in-going beam in the state ρ into an out-going beam in the state $\rho' = T\rho T^*$.

A linear, mixing (depolarizing) instrument transforms ρ instead into a sum of several terms of the form $T\rho T^*$.

It is therefore described by a superoperator,
a real 4×4 Mueller matrix acting on the Stokes vector,
or equivalently a completely positive linear map
on the space of 2×2 matrices.

A fully polarized beam (a pure polarization state)
has $p = 1$ and is described by a rank 1 density matrix,
which can be written in the form $\rho = \psi\psi^*$
with a state vector ψ determined up to a phase.

In this case, the intensity of the beam is $S_0 = |\psi|^2 = \psi^*\psi$.

A linear, non-mixing (not depolarizing) instrument
with Jones matrix T transforms
an in-going beam in the pure state with state vector ψ into
an out-going beam in a pure state with state vector $\psi' = T\psi$.

If the instrument is lossless, the intensities of
the in-going and the out-going beam are identical.

This is the case if and only if the Jones matrix T is unitary.

Passage through inhomogeneous media can be modelled
by means of slices consisting of many very thin filters
with Jones matrices close to the identity.

If Δt is the time needed to pass through one slice and $\psi(t)$ denotes the pure state at time t then $\psi(t+\Delta t) = T(t)\psi(t)$, so that

$$i\hbar \frac{d}{dt}\psi(t) \approx \frac{i\hbar}{\Delta t}(\psi(t+\Delta t) - \psi(t)) = \frac{i\hbar}{\Delta t}(T(t) - 1)\psi(t).$$

In a continuum limit with

$$T(t) = 1 - \frac{i\Delta t}{\hbar} H(t) + O(\Delta t^2)$$

we thus recover the time-dependent **Schrödinger equation**

$$i\hbar \frac{d}{dt}\psi(t) = H(t)\psi(t).$$

The unitarity of $T(t)$ implies that $H(t)$ is Hermitian.

Thus a polarized monochromatic ray of classical light
behaves exactly like a modern qubit.

It is instructive to read STOKES' 1852 paper in this light.

One finds there all the modern quantum phenomena
for qubits, explained in classical terms.

Splitting polarized beams into two beams with different,
but orthogonal polarization corresponds to writing
a wave functions as superposition of preferred basis vectors.

Mixtures are defined (in his paragraph 9) as arising
from "groups of independent polarized streams"
and give rise to partially polarized beams.

What is now the density matrix is represented by STOKES
with four real parameters, in today's terms comprising
the Stokes vector.

STOKES asserts (in his paragraph 16) the impossibility
of recovering from a mixture of several distinct pure states
any information about these states beyond what is encoded
in the Stokes vector (i.e., the density matrix).

The latter can be linearly decomposed in many
essentially distinct ways into a sum of pure states,
but all these decompositions are optically indistinguishable.

2. What exists in a beam of light?

Many quantum experiments start with a source producing a
well-collimated, monochromatic, unpolarized beam of light.
Such a beam is always stationary – nothing happens in the beam.
It is simply constant in time, apart from an unobservable phase.
(Only relative phases are observable.)

A sun ray shining on a sunny day through a small
(but not too small) hole in a dark room
with a specially colored window (acting as a chromatic filter),
filled with dusty air, gives an intuitive picture of such a beam.

The dusty air only serves to make the beam visible.
If one clears the air, one can only see the hole
and the colored spot on the wall.

From a knowledge of geometric optics and the location
of hole and spot, one can reconstruct the direction of the beam
by drawing a line between the hole and the spot.
The beam is still there but cannot be seen.

But is it really there?

According to the Copenhagen interpretation,
apparently not, since it is not observed.

To observe the beam, one needs the dust,
and a person in the room to notice the reflected light.

*"No elementary phenomenon is a phenomenon
until it is an observed phenomenon"* (Wheeler)
is a frequently repeated quantum credo.

But light had been studied centuries before the
Copenhagen interpretation.

No one had had any difficulty with the idea that the beam
is objectively there, with the definite properties studied
in classical optics, namely the complex amplitude
or the two real quadratures of the beam.

These properties could always be reproducably tested by putting
objects (mirrors, filters, prisms, etc.) into the path of the beam
and observing the results of the interaction.

Although this of course destroys the observed part of the beam
(which is analogous to an alleged 'collapse of the wave function'),
it had never been interpreted as a proof that the beam
has objective properties only when we observe it,
or as an indicator that the state of the beam is
not objectively determined (though initially unknown).

3. The position of quantum objects

The somewhat spooky view of the Copenhagen interpretation
stems basically from Heisenberg's uncertainty relation.

This relation implies that it is impossible
to assign consistently a sharp position
and a sharp momentum to a quantum object.

To account for this, the Copenhagen interpretation
simply denies quantum objects to have definite properties
– except in the special case where the state
is an eigenstate of the corresponding operator.

This may have been reasonable in the old days
where particles could only be observed collectively.

But nowadays there are many experiments
done routinely with single quantum objects.

Quantum mechanics is very successfully applied to the analysis of single quantum objects, whether they are electrons in a quantum dot, atoms in an ion trap, or big objects like a buckyball or the sun.

On the other hand, the interpretation of quantum mechanics
is conventionally based on the assumption that
quantum measurements are done only on an
ensemble of similarly prepared systems. Strictly speaking,
the traditional statistical interpretation even requires that
these similarly prepared systems are stochastically independent.

Thus the traditional foundations appear to be
inadequate for modern quantum mechanics.
There seems to be a discrepancy between the practice
of quantum mechanics and its traditional foundations.

We need a language for quantum mechanics that allows us
to speak of the position (and other properties)
of single quantum objects at least in an approximate way.

There is no doubt that to some limited accuracy,
quantum objects have a definite position – they are located
under this atom microscope, in that quantum dot, etc.,
and this implies knowledge of an approximate position.

It also implies knowledge of an approximate momentum
– they are roughly at rest in the laboratory frame.

In which sense is this compatible with a quantum description?

Clearly, the uncertainty relation is not violated.

In some sense, this approximate position and momentum
are a mean position and a mean momentum.

But the mean of what?

Traditional quantum mechanics does not answer this.
But it provides formulas for the **computation**
of the mean position $\langle q \rangle$ and the mean momentum $\langle p \rangle$
of each quantum object which can be prepared
as an **individual** ...

... provided that one assigns a state to each individual object.

Those strictly adhering to a statistical interpretation
may find this a forbidden use.

But how else shall we encode into quantum mechanics the
knowledge that, at a particular time, a particular object
is at a particular place in the experimental setup?

4. Making sense of fuzzy concepts

Photon particles (as discussed in the first companion lecture) have – both in their classical and quantum version –

- a fuzzy shape (boundary not clearly defined by fixing the total energy),
- a fuzzy internal structure (of a wave packet),
- a fuzzy but approximately defined position,
- a fuzzy but approximately defined time of passing an instrument.
- and (at least in case of photons on demand) a nearly deterministic behavior.

What holds for photon particles (i.e., localized wave packets
of a classical or quantum electromagnetic field),
holds more generally.

The position of **any** extended object is uncertain
within some limit resolution related to the diameter of the object.

For example, the limit resolution with which one can determine
the longitude and latitude of a city such as Vienna
is bounded by about 10–20 km.

Thus it is unreasonable to require that every well-defined object
has a position specified up to the 100th significant digit.

Physical concepts must be such that they
describe the object well at the resolution of interest.

Statistics is the traditional language for
the description of uncertainty.

But the probabilistic interpretation of statistics
is not always adequate.

In quantum mechanics we can compute
the "mean position" of a single particle.

But we have no idea **of what** this should be the mean.

Following the exposition of statistical thermodynamics
in the book Neumaier & Westra 2008, we therefore
introduce formal statistical mathematics, but phrased
in a way not using probabilistic terminology.

In thermodynamics, all we ever measure are macroscopic values
of the form $\langle g \rangle := \operatorname{tr} \rho g$, where g is a microscopic quantity
and ρ is the density matrix encoding the state of the system.

With some care, this **thermal interpretation**
can be extended to the microscopic regime in a way
that it remains sensible even for single, small quantum objects.

Instead of calling $\langle g \rangle := \text{tr } \rho g$ the expectation value,
we continue to call it the **value** of g,
as in the thermal case.

For a Hermitian quantity g with nonzero value $\langle g \rangle$,
we define the **limit resolution**

$$\text{Res}\,(g) := \sqrt{\langle g^2 \rangle / \langle g \rangle^2 - 1}.$$

The expression under the square root is always nonnegative,
and vanishes in a pure state iff it is an eigenstate of g.

Therefore, the limit resolution is an uncertainty measure
specifying how accurately one can treat g as a sharp number,
given by this value.

In experimental practice, the limit resolution is a lower bound on the relative accuracy with which one can expect $\langle g \rangle$ to be determinable reliably from (ideal) measurements of a single object at a single time.

In particular, a quantity
g is considered to be **significant** if $\mathrm{Res}\,(g) \ll 1$,
while g is considered to be **noise** if $\mathrm{Res}\,(g) \gg 1$.

If g is a quantity and $\widetilde{g}$ is a good approximation of its value then $\Delta g := g - \widetilde{g}$ is noise.

For a single system at a single time,
the values $\langle g \rangle$ have an experimentally testable meaning
precisely when $\mathrm{Res}\,(g)$ is significantly smaller than 1.

However, time averages of time-dependent values $\langle g(t_l) \rangle$
at different times t_l may have a testable meaning:
They are the values of the quantity $\tilde{g}$ defined as the average
of the $g(t_l)$. By the law of large numbers, this quantity
may be significant in the above precise sense
even when no single $g(t_l)$ is.

This terminology captures correctly the experimental practice,
without imposing any statistical or probabilistic connotations.

On the contrary, it determines the precise conditions
under which statistical reasoning is necessary:
namely precisely when the limit resolution of a quantity
is larger than the desired accuracy.

5. Probability via expectation

Kolmogorov 1933 gave the modern axiomatic foundations of (classical, commutative) probability theory, impeccably basing the concept of random variables on measure theory.

There is a nice book by Peter Whittle 1992 with the title "Probability via Expectation", who proceeds in a different way.

He starts with expectation, and **derives**
the measure theoretic approach rather than postulating it.

This has important advantages since the measure theoretic approach is limited to classical physics, where all quantities commute.

In quantum mechanics, noncommuting pairs of quantities,
such as position and momentum, defy a probabilistic description
in measure theoretic terms.

The probability interpretation is restricted to families
of pairwise commuting quantites X_j determined
by an experimental set-up.

Probabilistic foundations of quantum mechanics are therefore
intrinsically muddled up with the problem of measurement.

But measurement cannot be fundamental since what constitutes
a good measurement is itself a complicated question
dependent on quantum theory and statistical mechanics.

This might explain the unsatisfactory state (Schlosshauer 2005)
of the foundations of quantum mechanics even over 80 years
after its mathematical basis was established.

The thermal interpretation together with Whittle's approach gives a convenient alternative to the probabilistic foundations.

Indeed, if the X_j are pairwise commuting quantum observables, Whittle's (classical) development implies that there is a way consistent with Kolmogorov to define for any quantum state random variables H and X_j such that the expectation of all sufficiently regular functions $f(H, X)$ defined on the joint spectrum of (H, X) agrees with the value of f.

Thus, in the pairwise commuting case, it is always possible to construct a probability interpretation for the quantities, *no matter whether or not some sort of microscopic or submicroscopic structure is assumed to exist.*

Even without a postulated probabilistic structure,
quantum states can be given a frequentist interpretation
in a context where many repeated experiments are feasible.

Then $\langle g \rangle$ is the **expectation value** of g,
empirically defined as an average over many realizations.

In this case, the limit resolution $\mathrm{Res}\,(g)$ becomes
the standard deviation of g, divided by the absolute value
of the expectation.

Therefore, when the frequentist interpretation applies,
the limit resolution measures the relative accuracy
of the realizations.

Thus the thermal interpretation of the
quantum mechanical formalism in terms
of the value $\langle g \rangle$ of a quantity g
and its limit resolution Res g
gives both a nonstatistical fuzzy view
applicable to single quantum objects,
and an additional statistical view applicable
when frequent repetition is feasible.

New aspects appear when we consider quantum fields,
needed for the understanding of photons
in terms of quantum electrodynamics.

6. What is an ensemble of fields?

Quantum field theory poses a special problem
for the foundations of quantum mechanics.

In a full QED (i.e., space-time) treatment of light,
one needs to make an operational definition of how to interpret
quantum expectations when only **one** space-time field exists,
rather than an ensemble of independently prepared fields.

The thermal interpretation discussed above
avoids the need to consider an ensemble.

However, the classical treatment of unpolarized light
provides a clue to a more specific interpretation
which will turn out to be very useful.

In the first 464 pages of their well-known, comprehensive book
Optical Coherence and Quantum Optics,
Mandel & Wolf 1995 deduce from the Maxwell equations
the properties of classical light, including polarization
and the (semi)classical photo effect.

Knowing already that polarization is a quantum phenomenon,
we expect that their treatment gives insight also useful
for quantum field theory.

Unpolarized light cannot be modelled
directly by the Maxwell equations.

Instead, within Maxwell's theory, the description
of unpolarized light is an effective description
of observable degrees of freedom of very complex
"fluctuating" solutions of the Maxwell equations.

This effective description is obtained by short-time averaging,
over a time short with respect to the time resolution
of the observations but long with respect to the time
light needs to complete one fluctuation.

In this effective description, the short-time average
of the fields themselves is essentially zero, while
the short-time average of quadratic expressions in the field
are nonzero, varying slowly enough to be observable.

The short-time average introduces naturally
a fuzzy element into the description.

This leads to the effective observables $\langle f \rangle$, defined as
the short-time average of a microscopic quantity f
described in terms of an exact solution
of the Maxwell equations.

Thus the ensemble (if one can still speak of one)
is formed by the uncertainty in the fuzzy nature
of the time of preparation or measurement, rather than
by repeated independent preparation of the field.

Formally, one can take a limit where the fluctuations
are infinitely fast and the short-time average extends
over infinitely short times.

Then one arrives at a stochastic version of the Maxwell equations,
where the fluctuating fields are described as random fields
satisfying the Maxwell equations, and observables are the
expectations of functionals of the fields.

Note that in this limit, the actual observables
are not the random fields f themselves
but their expectations $\langle f \rangle$!

Random space-time fields which are stationary in time
give models for beams of partially polarized light.

Their coherence matrices provide the microscopic basis
for the macroscopic description of beams of light
in terms of the density matrix or the Stokes vector.

On the basis of the Maxwell equations and their stochastic
interpretation, the second order coherence theory
fully explains all polarization effects in a classical way.
(The photoeffect ist already explained
by the deterministic version.)

7. Classical quantum computing

One of the potential applications of
quantum information theory is quantum computing.
It heavily exploits entanglement properties of quantum systems.

Since entanglement also exists in classical field theories,
we now inquire about consequences for potential
applications of entanglement to quantum computing.

Remarkably enough, recent years have shown
that the speed-up achievable by quantum computations
can also be obtained by computing with classical fields,
at least for database search.

The speed-up from $O(N)$ to $O(\sqrt{N})$ in searching for
one of N items is not an intrinsic quantum feature
but can be reproduced classically. See

Bhattacharya et al. 2002, Lee 2002,
Bassi & Ghirardi 2003, Patel 2006,
Romanelli & Donangelo 2006.

The classical algorithm is easier to implement:
Bhattacharya et al. 2002 implemented $N = 32$ at a time
when quantum computers could only do $N = 4$.

Quantum versions may still have a space advantage for large N
since, in principle they can be implemented
in space of size $O(\log N)$ using multiphoton entanglement,
while the classical wave algorithm needs space $O(N)$.

8. Simulating quantum mechanics

The simulation of quantum computing by classical fields
is essentially achieved by using an optical network
in which each quantum level is modelled by a
corresponding mode of the electromagnetic field.

The linearity of the Maxwell equations then directly translates
into the superposition principle for pure quantum states.

Thus it is possible to **simulate arbitrary quantum systems**
which have a **finite** number of levels
by the Maxwell equations, and hence by a classical model.

Therefore we shall look a little more closely into the
reasons for this ability to simulate quantum systems.

The second order coherence theory of the Maxwell equations gives a valid description not only for classical beams of light but for general classical electromagnetic fields in the vacuum (or, with appropriate modifications, in other media).

The relevant effective observables for classical e/m fields are the momentum-dependent 3×3 coherence matrices

$$C(\mathbf{p}_1, \mathbf{p}_2, t) := \langle \mathbf{A}(\mathbf{p}_1)\mathbf{A}(\mathbf{p}_2)^* \rangle_t,$$

where $\mathbf{A}(\mathbf{p})$ is the spatial Fourier transform of $\mathbf{A}(\mathbf{x}, 0)$.

They satisfy the transversality conditions

$$\mathbf{p}_1^T C(\mathbf{p}_1, \mathbf{p}_2, t) = 0, \quad C(\mathbf{p}_1, \mathbf{p}_2, t)\mathbf{p}_2 = 0,$$

and completely determine all short-time averages of second order expressions in the fields.

The $C(\mathbf{p}_1, \mathbf{p}_2, t)$ describe Fourier modes of local intensities with frequencies $\omega = (|\mathbf{p}_2| - |\mathbf{p}_1|)c/\hbar$, with a dynamics given by

$$i\hbar \frac{d}{dt} C(\mathbf{p}_1, \mathbf{p}_2, t) = c(|\mathbf{p}_2| - |\mathbf{p}_1|) C(\mathbf{p}_1, \mathbf{p}_2, t).$$

We may interpret $C(\mathbf{p}_1, \mathbf{p}_2, t)$ as the kernel of the time-dependent operator $C(t)$ defined by

$$(C(t)\psi)(\mathbf{p}) := \int d\mathbf{p}' C(\mathbf{p}, \mathbf{p}', t)\psi(\mathbf{p}')$$

on the Hilbert space

$$\mathbb{H} = \{\psi \in L^2(\mathbb{R}^3, \mathbb{R}^3) \mid p^T \psi(p) = 0\}.$$

The dynamics can then be written as an operator equation

$$i\hbar \frac{d}{dt} C(t) = [H, C(t)],$$

where $H = c|\mathbf{p}|$ is the kinetic energy of a massless relativistic particle with momentum $\mathbf{p}$.

Thus the $C(t)$ satisfy naturally the **Heisenberg equation**

$$i\hbar \frac{d}{dt} C(t) = [H, C(t)].$$

with the free Hamiltonian $H = c|\mathbf{p}|$, a dynamical equation otherwise only known from quantum theory.

It is also clear that $C(t)$ maps transversal wave functions ψ with $\mathbf{p} \cdot \psi(\mathbf{p}) = 0$ into transversal wave functions $C(t)\psi$.

Thus the classical second order coherence theory
of the Maxwell equations is fully equivalent with
the quantum theory of a transverse vector particle
with kinetic energy $H = c|\mathbf{p}|$ in the momentum representation.

It is therefore capable of faithfully representing
all quantum properties of a **single** photon.

We conclude that

classical second-order stochastic optics is precisely the quantum mechanics of a single photon, with all its phenomenological bells and whistles.

As a consequence, it is possible (at least in principle)
to simulate with classical electromagnetic waves
and suitable classical linear optical networks
any quantum system that can be embedded
into the single photon quantum system.

Since all Hilbert spaces arising in applications
of quantum physics are separable, they have
a countable basis, and can be embedded into the
single photon quantum system, at least in principle.

Thus it appears that, **all** quantum systems
can be simulated by classical electromagnetic waves!

Of course, a practical realization may be difficult.

We are now nearly ready to put the puzzle together
whose pieces I have shown to you.

We have looked at quantum mechanics
from many different perspectives,
everywhere uncovering its classical aspects.

All this suggests the existence of a classical theory
underlying quantum mechanics.

However, we need to overcome one further obstacle.

In our picture there is still something missing ....

But for this lecture, it is a good time to stop.

The third and final part of the story will soon be online at

http://www.mat.univie.ac.at/~neum/papers/physpapers.html#detslides

References are given below.

References

[1] A. Aiello, G. Puentes, and J. P. Woerdman. Linear optics and quantum maps. Physical Review A 76 (2007), 032323. quant-ph/0611179.

[2] A. Bassi and G. Ghirardi, Quantum vs. classical computation: a proposal opening a new perspective, Manuscript (2003). quant-ph/0304074

[3] N. Bhattacharya et al., Implementation of quantum search algorithm using classical Fourier optics, Phys. Rev. Lett 88 (2002), 137901.

[4] D. Bohm, A suggested interpretation of the quantum theory in terms of 'hidden' variables, I and II, Phys. Review 85 (1952), 166–179, 180–193. Reprinted in [22].

[5] A. Fresnel, Considérations mécaniques sur la polarisation de la

lumière, 1866. Reprinted and translated into English in [21].

[6] C. Huygens, Treatise on light, 1690, Dover Publications, 1962. The relevant parts are reprinted in [21].

[7] R.C. Jones, A new calculus for the treatment of optical systems, J. Opt. Soc. Amer. 31 (1941), 488–493. Reprinted in [21].

[8] K.F. Lee, Classical Wave Simulation of Quantum Measurement, Ph.D. Thesis, 2002.

[9] U. Leonhardt and A. Neumaier, Explicit effective Hamiltonians for general linear quantum-optical networks, J. Optics B: Quantum Semiclass. Opt. 6 (2004), L1–L4. quant-ph/0306123

[10] E.-L. Malus, Sur une propriété de la lumière réfléchie, Mém. Phys. Chim. Soc. D'Arcueil 2 (1809), 143–158. Reprinted and partially translated into English in [21].

[11] L. Mandel and E. Wolf, Optical Coherence and Quantum Optics, Cambridge University Press, 1995.

[12] A. Neumaier, On a realistic interpretation of quantum mechanics, Manuscript (1999). quant-ph/9908071

[13] A. Neumaier, Bohmian mechanics contradicts quantum mechanics, Manuscript (2000). quant-ph/0001011

[14] A. Neumaier, Ensembles and experiments in classical and quantum physics, Int. J. Mod. Phys. B 17 (2003), 2937–2980. quant-ph/0303047

[15] A. Neumaier and D. Westra, Classical and Quantum Mechanics via Lie algebras, Book manuscript (2008). Cambridge University Press, to appear. arXiv:0810.1019

[16] A.D. Patel, Wave algorithms: Optimal database search and catalysis, Int. J. Quantum Information 4 (2006), 815–825. quant-ph/0609042

[17] H. Poincaré, Concerning theoretical descriptions of polarized light, Théorie mathématique de la lumière, Paris 1892. Reprinted in [21].

[18] A. Romanelli and R. Donangelo, Classical search algorithm with resonances in $\sqrt{N}$ cycles, Manuscript (2006). quant-ph/0608019

[19] M. Schlosshauer, Decoherence, the measurement problem, and interpretations of quantum mechanics, Rev. Mod. Phys. 76 (2005), 1267-1305. quant-ph/0312059

[20] G.G. Stokes, On the composition and resolution of streams of polarized light from different sources, Trans. Cambridge Phil. Soc. 9 (1852), 399–416. Reprinted in [21].

[21] W. Swindell, Polarized light, Dowden 1975.

[22] J.A. Wheeler and W. H. Zurek, Quantum theory and measurement. Princeton Univ. Press, Princeton 1983.

[23] N. Wiener, Generalized harmonic analysis, Acta Math. 55 (1930), 182–195.