Three lectures on coherent quantization and field theory

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Lectures given December 11–18, 2023 at the University of Erlangen, Germany

The first two lectures (December 11 and 14) can be followed independently. The third lecture is based on the first two.

For abstracts, slides and preprints (once available) see
`https://arnold-neumaier.at/cohErlangen2023.html`

For details on the mathematics see
`https://arnold-neumaier.at/cohSpaces.html`

Coherent Quantization I:
Coherent spaces for linear fields

Monday, December 11, 2023, 14:15-15:45, AG Lie-Gruppen, Übungsraum Ü2, 01.251, Cauerstr. 11, Erlangen

Coherent Quantization II:
Causal groups and quantum fields

Thursday, December 14, 2023, 16:15-18:00, AG Mathematische Physik, Übungsraum Ü1, 01.250, Cauerstr. 11, Erlangen

Coherent Quantization III:
Infra Fock spaces and nonlinear fields

Monday, December 18, 2023, 14:15-15:45, AG Lie-Gruppen, Übungsraum Ü2, 01.251, Cauerstr. 11, Erlangen

The origin of coherent quantization

Let us begin at the origins....

In the beginning God created the heavens and the earth.
And God said, "Let there be light", and there was light.
(Genesis 1:1.3)

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As we now know, it took Him only 10 seconds:

The photon epoch started after most leptons and anti-leptons were annihilated at the end of the lepton epoch, about 10 seconds after the Big Bang. (`https://en.wikipedia.org/wiki/photon_epoch`)

Today, the photons in ordinary light are modelled in terms of **coherent states** (SCHRÖDINGER 1926) – directly related to the present lectures.

For us mortals, understanding the Creation is a fascinating but much slower process. For me, part of the fascination is the discovery of the mathematical structure of physics.
It may be viewed as part of God's thoughts when He designed the universe.

Even almost a century after Schrödinger, our understanding of quantum mechanics is still limited, with important unsolved problems in

- the foundations of quantum mechanics (measurement problem),
- the foundations of quantum field theory (**does QED exist?**),

and

- quantum gravity (what is it, really?)

The last two problems can be phrased as **construction problems for certain infinite-dimensional coherent manifolds**.

The **Wightman axioms** describe relativistic quantum field theories with a unique vacuum state.

In 4 space-time dimensions, only free and quasifree quantum field theories satisfying the Wightman axioms are known.

The question of the existence of interacting relativistic quantum field theories in 4 space-time dimensions is completely open.

The field theories satisfying Wightman axioms describe particle scattering.

Instead, we study field theories in my coherent quantum physics program from the general point of view of symmetry and causality.

Symmetry is described in terms of infinite-dimensional groups.

Causality (responses are later than their cause) is described by **causal spaces**.

Both merge in the concept of a **causal group**.

From the coherent perspective, the real dichotomy is not between free and interacting but between linear and nonlinear quantum fields.

Even classically, linear field theories are easy to construct, but nonlinear fields pose a much greater challenge.

Two of the 7 Clay millennium prize problems are associated with nonlinear fields:

– for classical fields, the global existence of solutions of the Navier–Stokes equations of fluid mechanics;

– for quantum fields, the existence of quantum field theories of Yang–Mills type.

In the second case, not even a precise mathematical formulation is available about which problem is to be solved; its modeling is part of the problem!

The Wightman axioms are now seen as too restrictive; for example, neither thermal (KMS) fields nor fields on curved spacetimes are covered.

The **Haag–Kastler axioms** for local quantum field theories are more general; but in 4 dimensional spacetime, again no nonlinear examples are known.

Coherent Quantization I:
Coherent spaces for linear fields (today)

This is the simple case. It introduces the coherent machinery and the beauty of (metalinear) groups.

Coherent Quantization II:
Causal groups and quantum fields (Thursday)

Defines the mathemaical causality concepts and relates them to dynamical C^*-algebras for relativistic quantum fields.

Coherent Quantization III:
Infra Fock spaces and nonlinear fields (next Monday)

Applies the new tools and paves the way towards quantum fluid mechanics.