Global Optimization – Integrating Convexity, Optimization, Logic Programming, and Computational Algebraic Geometry (GICOLAG)

Organizers: I. Emiris, I. Bomze, L. Wolsey, A. Neumaier (local organizer)

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Report on the program

Global optimization is the task of finding the absolutely best set of admissible conditions to achieve an objective under given constraints, assuming that both are formulated in mathematical terms. A special case is the constraint satisfaction problem, where one just wants to find one or all solutions of a given set of constraints. In terms of applications, global optimization is very interdisciplinary; with respect to methods, the last few years were characterized by increasing cross-fertilization with related subjects.

The program was designed to bring together key experts from the field of global optimization and the disciplines providing theoretical and algorithmic support for solving global optimization problems: convexity, optimization, logic programming, and computational algebraic geometry. The aim of the program was to emphasize these connections and to enhance the integration of the various techniques into a unified approach to global optimization, mixed integer nonlinear programming, and constraint satisfaction. The hope is that this will result in improved software for the solution of such problems, with resulting speed-ups (for industrial applications) and increased reliability (for safety-critical applications and computer-assisted proofs).

The emphasis was on algorithmic problems and their theoretical analysis but work on applications to robotics, engineering design, and other fields was also included. During the program 36 researchers from the United States of America, Japan, Algeria, and 11 European countries visited the ESI and interacted effectively.

A peak of the activities was during and around the GICOLAG workshop (December 4–8). The full program of the workshop is available at the GICOLAG website `http://www.mat.univie.ac.at/\~neum/gicolag.html`. Further activities involving scientific collaborations took also place during most of the remaining time; in particular, there were numerous additional lectures. A complete list of the talks given during the program, together with slides for most of the lectures presented, can be downloaded from the GICOLAG website.

The main topics discussed during the program were

- Large-scale local optimization
- Mathematically rigorous algorithms for verified computations
- Rigorous global optimization
- Global optimization of differential constraints
- Global quadratic optimization
- Discrete optimization
- Algorithmic algebraic geometry
- Integration frameworks
- Applications

In the following we summarize the highlights within each topic.

1. Large-scale local optimization

The well developed theory of local optimization plays a central role for global optimization; finding a local optimizer is an important step in solving the global problem and succeeds if the starting point is close enough to the global minimizer. In our program, the scientific exchange in this area concentrated on the study and discussion of methods that work for large problems, with a focus on interior point methods. There were also several talks on this topic (Jarre, Sartenaer, di Serafino).

2. Mathematically rigorous algorithms for verified computations

Most numerical computations are approximate only since most real numbers cannot be represented exactly on the computer. Successful algorithms for traditional tasks like linear programming and less traditional problems like semidefinite optimization need a careful implementation to provide guaranteed results in the presence of rounding errors, usually using techniques from interval analysis.

During the program there was much discussion of recent breakthroughs that allow one to use some algorithms in a black box fashion and still guarantee correct results by appropriate postprocessing. Based upon rigorous postprocessing work presented by Keil (for linear programming) and Jansson (for semidefinite optimization), good progress was made during the program for semi-infinite linear programming, where the uncountable number of constraints pose additional difficulties. It is expected that this will lead in the near future to rigorous software for semi-infinite linear programming, which will make many problems of approximation theory rigorously tractable on the computer.

Applications of interval analysis to computer-assisted proofs were discussed in a lecture by Csendes and in informal discussions centering around a polynomial problem by Rump.

3. Rigorous global optimization

Interval analysis is a perfect tool for automatically computing estimates on function values, derivatives, bounds on nonlinearities, thus enabling the rigorous numerical solution of global optimization problems. The successful application requires care, however, to avoid excessive overestimation that characterizes the naive approaches. This poses numerous mathematical problems that were discussed. Work in this direction was presented by Kearfott, Kolev, Lebbah, Neumaier, Rueher, and Rump. The many connections to the other topics of the programs were investigated, and ways to overcome limitations of the software packages presented in some of the talks were discussed.

4. Global optimization of differential constraints

Differential constraints pose a special challenge in rigorous global optimization, since the numerical solution of differential equations is very difficult to accompany by a rigorous error analysis, and optimization compounds the difficulties. New approaches were discussed, involving work by Adjiman, Ratschan, and Tucker. New theory for relaxations of differential constraints developed during the program should enable the creation of rigorous global solution of optimal control problems in the near future.

5. Global quadratic optimization

Quadratic optimization problems arise in many applications, either as the next modeling level beyond linear programming, or because of a combinatorial structure which often leads to a natural quadratic formulation. Because of its special structure, special mathematical techniques can be brought to bear on these problems. Work by Dür and Anstreicher pushed the limits forward on what can be achieved in this direction.

6. Discrete optimization

Because the finiteness of combinatorial optimization problems, many discrete problems are globally tractable. The challenge here is to push further away the limits where the combinatorial explosion (worst case exponential work) makes the actual computations infeasible. The known connections with semidefinte programming were further explored by Anstreicher and Bomze, using special problem structure. On the other hand, the sensitivity analysis of discrete problems is more difficult than in the continuous case since the notion of small deformations is absent. Interesting discussions centered around recent work of Hooker, utilizing decision diagrams for discrete sensitivity analysis.

7. Algorithmic algebraic geometry

The approximability of sets of polynomial constraints by convex constraints in a higher-dimensional representation led in the last few years to a flurry of activities relating algebraic techniques involving polynomials and global optimization. Progress in this area was reviewed and many of the possibilities that became visible in this direction were investigated. Even old problems concerning polynomials received new attention, and found improved solutions form a complexity point of view. Lectures by Emiris, Garloff, Mourrain, Tsigaridas, and Winkler contributed to this area.

8. Integration frameworks

An important goal of the program, and a center of attention was the integration of the various tools and techniques to improve the quality of current global optimization software. Several researchers (Domes, Fourer, Schichl, Vigerske) concentrated on this aspect, and presented integration frameworks on the level of modeling languages, software systems, and graphical user interfaces. In collaboration during the program, various software systems have been connected to work together, and have been ported to additional platforms.

A notable highlight achieved during the program was work which showed for the first time how to achieve the nearly rigorous global optimization of large-scale black box problems (where a strict mathematical guarantee is impossible by nature of the data available), based on Stefan Vigerske's branch and cut algorithm LaGO for nonconvex mixed-integer nonlinear programs.

9. Applications

Apart from its intrinsic mathematical interest, global optimization is important for its relevance in applications. Demanding applications from all sorts of areas pose new mathematical challenges to the field. Mainly three important application areas got the attention of the program: robotics, uncertainty modeling, and engineering design. Especially in robotics, global optimization methods have already made a significant impact; beautiful lectures by Jaulin and Merlet illustrated this, and at the same time gave rise to more difficult new problems. Uncertainty modeling gives rise to optimization problems tractable by semidefinte programming and global optimization. Engineering design often aims for optimality; example applications were presented by Gay, Markot and Vigerske. Other applications were discussed in lectures by Hochbaum (ranking teams or scientific proposals) and Hooker (decision support).

Invited Scientists: Claire Adjiman, Kurt Anstreicher, Immanuel Bomze, Tibor Csendes, Ferenc Domes, Mirjam Duer, Ioannis Emiris, Robert Fourer, Jürgen Garloff, David Gay, Dorit Hochbaum, John Hooker, Christian Jansson, Florian Jarre, Luc Jaulin, Baker Kearfott, Christian Keil, Lubomir Kolev, Yahia Lebbah, Mihaly Markot, Andy McLennan, Jean-Pierre Merlet, Bernard Mourrain, Masakazu Muramatsu, Arnold Neumaier, Stefan Ratschan, Franz Rendl, Michel Rueher, Siegfried Rump, Annick Sartenaar, Hermann Schichl, Elias Tsigaridis, Warwick Tucker, Stefan Vigerske, Franz Winkler, Laurence Wolsey.