STABILITY ANALYSIS OF LINEAR SYSTEMS VIA TRANSFORMING THE CHARACTERISTIC POLYNOMIAL MODEL TO THE EIGENVALUE PROBLEM UNDER INTERVAL UNCERTAINTIES

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Abstract: The paper addresses the stability analysis of linear continuous systems under interval uncertainties. This problem initially described by the characteristic polynomial is transformed into a corresponding eigenvalue problem. The final analysis is equivalent to estimating the eigenvalues of matrices which elements are nonlinear functions of interval parameters. A method for obtaining the exact range of the eigenvalues is used. It can be applied if certain monotonicity conditions are fulfilled. The method appeals to computing tight outer bounds on the eigenvalues range. The outer bounds are obtained as a solution of an algebraic nonlinear system. A numerical example, illustrating the applicability of the methods presented, is solved in the end of the paper. It is a linear system for fine position of object with big weight and inertness.

Key words: stability analysis of linear systems, eigenvalues of interval matrices with dependent coefficients, outer bounds and exact ranges.

1. INTRODUCTION

Lets have the linear continuous interval system described by the characteristic polynomial

$$q(s,p)=\sum_{i=0}^{n} a_i(p)s^{n-i}, \ \ p\in \boldsymbol{p} . \tag{1}$$

Remark 1: Vector p is the column-vector of independent system parameters but $\boldsymbol{p}$ is the respective vector with interval components.
Various interval criteria can be used for check the system stability ([2] – interval form of Raus criterion and [3] – interval form of Frazer-Duncan criterion). Both of them are based on interval extensions of nonlinear function which are calculated using generalized intervals and affine arithmetic. These interval criteria have some disadvantages.

1. They are insufficient effective for calculating the interval extensions studied in the cases with independent and dependent coefficients in characteristic polynomial (1). They take up a lot of resources but the results are a little better.
2. They take up a number of calculations when it analyses stability of the linear continuous system with known stability margin.
3. They use interval extensions which are conservative and already are wider than the exact ranges of interval functions studied.

In these reason, it is not suitable and not recommends applying the interval form of classic stability criteria in interval form when the linear continuous system is described by characteristic polynomial (1). Then, in this paper, we propose other approach for analyzing the linear continuous systems stability.

2. PROBLEM STATEMENT

It is well known that based on the characteristic polynomial (1) can be define the matrix

$$A_{nxn}=\begin{bmatrix} -\dfrac{a_1(p)}{a_0(p)} & -\dfrac{a_2(p)}{a_0(p)} & \dots & -\dfrac{a_{n-1}(p)}{a_0(p)} & -\dfrac{a_n(p)}{a_0(p)} \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}. \tag{2}$$

Then the system described by (1) is stable if and only if the eigenvalue of matrix A (see (2)) have negative real part.
We consider the following “perturbed” eigenvalue problem:

$$A(p)x=\lambda x, \ \ p\in \boldsymbol{p} . \tag{3}$$

Remark 2: Each matrix $A(p), \ p\in \boldsymbol{p}$ is assumed non-singular.

It follows from (3) that each eigenvalue λ and its corresponding eigenvector x are implicit forms of p.
Here, we are interested in the intervals of the eigenvalues of (3).
We will estimate only the interval of the maximum eigenvalue obtained by the (center, nominal) problem

$$A(p^0)x=\lambda x . \tag{4}$$

Remark 3: In general, the methods suggested later can be applied for any other real eigenvalues.
Let

$$\lambda_{\max}^0(p^0)=\max\left(\lambda_k(p^0)\right), \ k=1,\dots,n \tag{5}$$

is a maximum eigenvalue while $x_{\max}^0=\left[x_1^0, \ \ x_2^0, \ \ \dots \ \ x_n^0\right]^{\mathrm T}$ is the corresponding eigenvector. We make the following assumption (ensuring structural stability of the problem).

Assumption A1: Let $\lambda_{\max}(p)$ and $x_{\max}(p)$, corresponding to all $p\in \boldsymbol{p}$ remain real.

On account of Assumption A1, the range

$$\boldsymbol{\lambda}_{\max}^*=\{\lambda(p): p\in \boldsymbol{p}\} \tag{6}$$

is a real interval.

Remark 4: For simplicity, we shall henceforth drop the index max if maximum eigenvalue $\lambda_{\max}$.

Without any loss of generality we need a second assumption. If the pair $\left(x^0, \ \ \lambda^0\right)$ is the solution of (4) then

Assumption A2: We assume that the absolute value of the nth component $\left|x_n^0\right|$ of vector x^0 is the largest component of the other components, i.e.

$$\left|x_n^0\right|\ge\left|x_i^0\right|, \ i\ne n \tag{7}$$

Now x^0 is normalized through

$$x_n^0=1 . \tag{8}$$

Further, we require that (8) be also valid for

$$x_n(p)=1, \ p\in \boldsymbol{p} . \tag{8a}$$

We introduce the n-dimensional real vector

$$y = [y_1,\ y_2,\ \dots\ ,\ y_n]^{\mathrm{T}} \tag{9}$$

with
$$\begin{aligned} y_i &= x_i(p),\ \ i = 1,\dots,\ n-1 \\ y_n &= \lambda(p) \end{aligned} . \tag{10}$$

Using (10), the eigenvalue problem (3) is

$$\left| \begin{array}{l} a_{11}y_1 + a_{12}y_2 + \dots + a_{1(n-1)}y_{n-1} - y_n y_1 + a_{1n} = 0 \\ a_{21}y_1 + a_{22}y_2 + \dots + a_{2(n-1)}y_{n-1} - y_n y_2 + a_{2n} = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}y_1 + a_{n2}y_2 + \dots + a_{n(n-1)}y_{n-1} - y_n .1 + a_{nn} = 0 \end{array} \right. , \tag{11}$$

where $a_{ij} = a_{ij}(p),\ \ p \in \boldsymbol{p}$. The system (11) can be written as:

$$\breve{A}(p)y - y_n y + A_n(p) = 0 \ \ , \tag{11a}$$

where $\breve{A}(p)$ is the same as $A(p)$ except for the nth column which is zero, $A_n(p)$ is the nth column of $A(p)$. It is seen that system (11) is nonlinear only because of the products $y_n y_i$, $i = 1,\dots,n$. The solution of (11) is the set

$$S(\boldsymbol{p}) = \{y : \breve{A}(p)y - y_n y + A_n(p) = 0, \quad p \in \boldsymbol{p}\} . \tag{12}$$

The interval hull of $S(\boldsymbol{p})$ will be denoted $\boldsymbol{y}^*$ and $\boldsymbol{y}^*$ will be called exact range to (11). Any other $\boldsymbol{y}'$ such that $\boldsymbol{y}^* \subset \boldsymbol{y}'$ will be referred to as an outer bound to (11).

The present paper addresses the problem of determining the outer bounds and the exact range of the solution of (11). First, a direct method for computing a tight and cheap outer bound $\boldsymbol{y}'$ is presented in Section 3. It is based on the approach suggested in [4]. In Section 4, the exact range $\boldsymbol{y}^*$ to (11) is determined for the case when certain monotonicity conditions, regarding the derivatives of y_i with respect to p_j, are fulfilled. It is based on the use of the outer solution method from the previous section. An illustrative example is considered in Section 5. The paper ends up with concluding remarks in the last Section 6.

3. OUTER BOUNDS OF THE EXACT RANGE

The functions defined by (11) can be written in the following form:

$$\begin{aligned} f_i(p,y) &= \sum_{j=1}^{n-1} a_{ij}(p)y_j - y_n y_i - a_{in}(p),\ \ i = 1,\dots,\ n-1 \\ f_n(p,y) &= \sum_{j=1}^{n-1} a_{nj}(p)y_j - y_n - a_{nn}(p),\ \ p \in \boldsymbol{p},\ \ y \in \boldsymbol{y} \end{aligned} , \tag{13}$$

The interval hull of $f_i(p,y),\ i = 1,\dots,n$ is $\boldsymbol{S}_{f_i}(p,y)$, $p \in \boldsymbol{p},\ y \in \boldsymbol{y}$.

On account of the inclusion property

$$f_i(p,y) \in \boldsymbol{S}_{f_i}(p,y),\ \ p \in \boldsymbol{p},\ \ y \in \boldsymbol{y} \tag{14}$$

the linear interval forms of $\boldsymbol{S}_{f_i}(p,y)$ are:

$$\boldsymbol{L}_{ij}(p) = \sum_{k=1}^{m} \alpha_{ijk} p_k + \boldsymbol{g}_{ij}, \quad p_k \in \boldsymbol{p}_k . \tag{15}$$

From (14) it follows

$$a_{ij}(p) \in \boldsymbol{L}_{ij}(p) , \quad p \in \boldsymbol{p} . \tag{15a}$$

To find the outer bound of $y_n = \lambda(p),\ p \in \boldsymbol{p}$ we present the elements a_{ij} as linear functions of independent system parameters, i.e.

$$a_{ij}(p) = \sum_{k=1}^{m} \alpha_{ijk} p_k + \boldsymbol{g}_{ij}, \quad p_k \in \boldsymbol{p}_k . \tag{16}$$

We apply (16) to (11) and we get the system:

$$\begin{aligned} &\sum_{j=1}^{n-1}\sum_{k=1}^{m} [\alpha_{ijk} p_k + \boldsymbol{g}_{ij}] y_j - y_n y_i + \left(\sum_{k=1}^{m} \alpha_{ink} p_k + \boldsymbol{g}_{in}\right) = 0 \\ &\qquad\qquad i = 1,\dots,\ n-1 \\ &\sum_{j=1}^{n-1}\sum_{k=1}^{m} [\alpha_{njk} p_k + \boldsymbol{g}_{nj}] y_j - y_n + \left(\sum_{k=1}^{m} \alpha_{nnk} p_k + \boldsymbol{g}_{nn}\right) = 0 \end{aligned} . \tag{17}$$

We substitute the interval variables in (17) with

$$\boldsymbol{p}_k = p_k^0 + u_k ,\ \ \boldsymbol{y}_j = y_j^0 + v_j ,\ \ \boldsymbol{g}_{ij} = g_{ij}^0 + t_{ij} . \tag{18}$$

On account of (18) we get the following system

$$\left| \begin{array}{l} (a_{11}^0 - y_{\mathrm{n}}^0)v_1 + a_{12}^0 v_2 + \dots + a_{1,(n-1)}^0 v_{n-1} - y_1^0 v_n = B_1 \\ a_{21}^0 v_1 + (a_{22}^0 - y_n^0)v_2 + \dots + a_{2,(n-1)}^0 v_{n-1} - y_2^0 v_n = B_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{(n-1),1}^0 v_1 + a_{(n-1),2}^0 v_2 + \dots + (a_{(n-1),(n-1)}^0 - y_n^0)v_{n-1} - \\ - y_{(n-1)}^0 . v_n = B_{n-1} \\ a_{n1}^0 v_1 + a_{n2}^0 v_2 + \dots + a_{n,(n-1)}^0 v_{n-1} - v_n = B_n \end{array} \right. \tag{19}$$

where $y_j^0,\ \ j = 1,\dots,n$ is the solution of the system (17) for the centers p^0 of the interval vector $\boldsymbol{p}$ while the meaning of the remaining symbols are the same as [4].

Now system (19) can be written in a compact form

$$\tilde{A}_0 v = B \tag{20}$$

where $\tilde{A}_0$ is the real coefficient matrix in (17) for $p = p^0$. Let $C = \tilde{A}_0^{-1}$, thus, (20) can be written in the form:

$$\begin{aligned} v = CB = &-\sum_{k=1}^{m}\left(C\breve{G}_k\right)u_k - CT\breve{y}^0 - \left[\sum_{k=1}^{m}\left(C\breve{H}_k\right)u_k\right]v - \\ &- CTv + Cv_n\breve{v} - \sum_{k=1}^{m}\left(CH_k^n\right)u_k - CT^n \end{aligned} \tag{21}$$

where

$$\breve{G}_k = \left[\breve{G}_{ik}\right]^T = \sum_{j=1}^{n-1} \alpha_{ijk} y_j^0 \ ; T = [t_{ij}];\ H_k^n = \left[\alpha_{ink}\right]^{\mathrm{T}} ;$$

$$\breve{H}_k = [\breve{H}_{ik}]^T = \left[\sum_{j=1}^{n-1} \alpha_{ijk}\right]^{\mathrm{T}} ; i,j = 1,\dots,n;\ \ k = 1,\dots,m;$$

$\breve{y}^0$ and $\breve{v}$ are the same as vectors y^0 and v, respectively, expect for the last element which is now zero; T_n is the last column of matrix T. We note system (21) by the radii

$$r = d + Dr + |C| r_n \breve{r} , \tag{22}$$

with

$$d = \left| -\sum_{k=1}^{m}\left(C\breve{G}_k\right)u_k - CT\breve{y}^0 - \sum_{k=1}^{m}\left(CH_k^n\right)u_k - CT^n \right| , \tag{22a}$$

$$D = \left| -\left[\sum_{k=1}^{m}(C\breve{H}_k)u_k\right] - CT \right| . \tag{22b}$$

The matrix equation (22) is a nonlinear real value (noninterval) system of n equations of n unknowns r_i:

$$r_i = d_i + \sum_{j=1}^{n-1} D_{ij} r_j + r_n \sum_{j=1}^{n-1} |c_{ij}| r_j + |c_{in}| r_n, \; i = 1,..., n. \quad (23)$$

We solve system (23) for r_i and, based on the component r_n, the outer bounds of the maximum eigenvalue $\boldsymbol{y}_n^{'}$ is:

$$\boldsymbol{y}_n^{'} = y_0^n + [-r_n, \quad r_n]. \quad (24)$$

The main result of this section is the theorem.

THEOREM 1: Assume the solution r of system (23) is positive. Then the outer bound $\boldsymbol{\lambda}_{\max}^{'}$ on the range $\boldsymbol{\lambda}_{\max}^{*}$ of the maximum eigenvalue λ(p) of (3) when $p \in \boldsymbol{p}$ is

$$\boldsymbol{\lambda}_{\max}^{'} = \boldsymbol{y}_n^{'} = y_n^0 + \boldsymbol{r}_n^{'} \quad (25)$$

where

$$\boldsymbol{r}_n^{'} = [-r_n, r_n] \quad (25a)$$

This theorem is valid for all the eigenvalues but to simplify the presentation we formulate the theorem only for the maximum eigenvalue.

Remark 5: The proof of the above theorem is the same as the proof of Theorem 2.1 in [4].

Thus, it has been shown that the problem of finding an outer bounds $\boldsymbol{\lambda}_{\max}^{'}$ on the range $\boldsymbol{\lambda}_{\max}^{*}$ reduces to solving the non-linear (incomplete quadratic) system (23). Since system (23) is only mildly non-linear, because of the products $y_n y_i \;, i = 1,...,n$, its solution does not present any problem.

4. THE EXACT RANGE

In this section, the outer bounds on the solution of system (11) will be applied in a method for computing the interval hull (exact range) $\boldsymbol{y}^*$. It is assumed that $a_{ij}(p)$ are continuously differentiable functions in p. The method suggested is applicable only if certain monotonic conditions are fulfilled and when the coefficients in the system (11) are dependent.

We are interested in expressing the derivative of y_i with respect to p_l, $i = 1,...,n; \; l = 1,...,m$. With this in mind, we differentiate (11) in p_l and get the system

$$\begin{aligned} &\sum_{j=1}^{n-1} a_{ij} \frac{\partial y_j}{\partial p_l} - \frac{\partial y_n}{\partial p_l} y_i - y_n \frac{\partial y_i}{\partial p_l} = -\sum_{j=1}^{n-1} \eta_{ijl} y_j + \eta_{inl} \\ &\sum_{j=1}^{n-1} a_{nj} \frac{\partial y_j}{\partial p_l} - \frac{\partial y_n}{\partial p_l} = -\sum_{j=1}^{n-1} \eta_{njl} y_j + \eta_{nnl}, \; i = 1,...,n\text{-}1 \end{aligned} \quad (26)$$

where

$$\begin{aligned} &\eta_{ijl} = \frac{\partial a_{ij}}{\partial p_l}, i, j = 1,..., n; \; l = 1,..., m \\ &\eta_{njl} = \frac{\partial a_{nj}}{\partial p_l}, \; j = 1,..., n; \; l = 1,..., m \;. \end{aligned} \quad (27)$$

We solve the system (26) using the method proposed in Section 3 to determine the outer bounds $\boldsymbol{D}_{il}$ of the derivatives

$$\frac{\partial y_i}{\partial p_l} = D_{il}, \quad p \in \boldsymbol{p}. \quad (28)$$

Next we will make the following assumption:

Assumption A3: We assume that each estimation $\boldsymbol{D}_{il}, \; i = 1,...,n, \; l = 1,...,m$ satisfies either the condition

$$\boldsymbol{D}_{il} \geq 0 \quad (29)$$

or

$$\boldsymbol{D}_{il} \leq 0 \;. \quad (30)$$

On account of inclusion property (29) the fulfillment of Assumption A3 guarantees that y_i is monotonic with respect to each p_l. Now we define two vectors as follows

$$\underline{p}_l^{(i)} = \begin{cases} \underline{p}_l^{(i)}, \boldsymbol{D}_{il} \geq 0, l = 1,..., m \\ \overline{p}_l^{(i)}, \boldsymbol{D}_{il} \leq 0, l = 1,..., m \end{cases}, i = 1,..., n, \quad (31a)$$

$$\overline{p}_l^{(i)} = \begin{cases} \overline{p}_l^{(i)}, \boldsymbol{D}_{il} \geq 0, l = 1,..., m \\ \underline{p}_l^{(i)}, \boldsymbol{D}_{il} \leq 0, l = 1,..., m \end{cases}, \; i = 1,..., n. \quad (31b)$$

We get the exact range $\boldsymbol{y}^*$ of (11) using the following theorem.

Theorem 2: If Assumption A3 holds for all $i = 1,...,n$ then the nth component $\boldsymbol{y}_n^* = (\underline{y}_n^*, \overline{y}_n^*)$ of the solution vector $\boldsymbol{y}^*$ is determined as follows:

1) $\underline{y}_n^*$ is equal to the nth component of the system solution:

$$\begin{aligned} &\sum_{j=1}^{n-1} a_{ij}(\underline{p}) y_j - y_n y_i + a_{in}(\underline{p}) = 0, \; i = 1,..., n-1 \\ &\sum_{j=1}^{n-1} a_{nj}(\underline{p}) y_j - y_n + a_{nn}(\underline{p}) = 0. \end{aligned} \quad (32a)$$

2) $\overline{y}_n^*$ is equal to the nth component of the system solution:

$$\begin{aligned} &\sum_{j=1}^{n-1} a_{ij}(\overline{p}) y_j - y_n y_i + a_{in}(\overline{p}) = 0, \; i = 1,..., n-1 \\ &\sum_{j=1}^{n-1} a_{nj}(\overline{p}) y_j - y_n + a_{in}(\overline{p}) = 0. \end{aligned} \quad (32b)$$

5. NUMERICAL EXAMPLE

The system studied is calibration device for high accuracy acceleration transducers. This requires working over an object, whose mechanical conditions should be close to those of an inertial body. Generated vibrations in the surrounding ground from heavy mechanical factories located near the object studied can be considered as a disturbance.

Figure 1 gives a physical model of the system studied. The bottom platform P is supported at each corner by a set of three elements lying on the ground: a spring, a damper and an electromagnetic force generator. Another platform B, bearing the calibration device, is leant on the first one through similar mechanical elements, except for the absence of active generators.

Two main simplifying hypotheses are assumed in order to obtain a suitable lumped parameter model of reasonable complexity:

1. The ground and platforms P and B are considered rigid bodies. Perfect symmetry of the structure and only vertical motion of the ground are assumed.
2. The four electromagnetic actuators at the corners of the lower platform P are driven by the same electric current, according to the hypothesis that perfect symmetry gives rise to translation motion of the object along the vertical axis only.

The simplified model of system studied is shown on the figure 2.

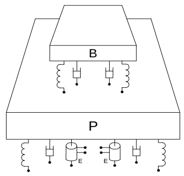

Figure 1

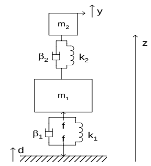

Figure 2

There i is command variable; z is position along vertical axis, y is controlled output and d is disturbance.

The structural scheme of closed system studied is:

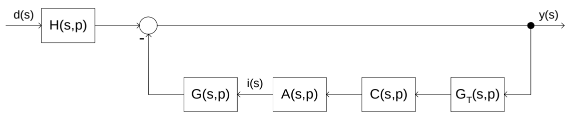

Figure 3

The respective transfer functions are: $A(s) = k_A$,

$G_T(s) = k_T s(1+s\tau)^{-1}$, $G(s) = N_G(s)D^{-1}(s)$, $H(s) = N_H(s)D^{-1}(s)$

where $N_G(s) = k_F k_1^{-1} s^2 (1+\beta_2 k_2^{-1} s)$, $N_H(s) = (1+\beta_1 k_1^{-1} s)(1+\beta_2 k_2^{-1} s)$

$$D(s) = m_1 m_2 k_1^{-1} k_2^{-1} s^4 + (m_1 \beta_2 + m_2(\beta_1+\beta_2)) k_1^{-1} k_2^{-1} s^3 +$$
$$+ (m_1 k_1^{-1} + m_2 (k_1^{-1} + k_2^{-1}) + \beta_1 \beta_2 k_1^{-1} k_2^{-1}) s^2 + (\beta_1 k_1^{-1} + \beta_2 k_2^{-1}) s + 1$$

The full vector of parameters is 10th dimensional : $p = \begin{bmatrix} k_A, & k_T, & k_F, & \tau, & k_1, & k_2, & m_1, & m_2, & \beta_1, & \beta_2 \end{bmatrix}^T$. For simplicity of the calculation and without any loss of generality only 3 of them can be considered as intervals (see Table 1).

			Center	Radius
k_1	N/m	p_1	14000	3000
k_2	N/m	p_2	10000	3000
β_1	Ns/m	p_3	480	100

Table 1

The values of other parameters are shown in Table 2.

		Value
k_A	A/V	8.7
k_T	Vs^3/m	2*10^5
k_F	N/A	10
τ	s	2
m_1	kg	4250
m_2	kg	440
β_2	Ns/m	1.7*10^4

Table 2

The characteristic polynomial of system considered is:

$$q(s,p) = a_0 s^5 + a_1 s^4 + a_2 s^3 + a_3 s^2 + a_4 s + a_5 \quad (33)$$

where

$$\begin{aligned}
a_0 &= 187 \\
a_1 &= 80869 + 0.044 p_3 \\
a_2 &= 4.064*10^7 + 0.044 p_1 + 2.556 p_2 + 10.50 p_3 \\
a_3 &= 3.638*10^9 + 10.50 p_1 + 2389.7 p_2 + 340 p_3 + 10^{-4} p_2 p_3 \\
a_4 &= 88.74*10^9 + 340 p_1 + 2.139*10^5 p_2 + 10^{-4} p_1 p_2 + 0.020 p_2 p_3 \\
a_5 &= 5.218*10^6 p_2 + 0.020 p_1 p_2
\end{aligned} \quad (33a)$$

The respective matrix to (40) is

$$A = \left[aa_{ij}\right]_{i,j=1,\dots,5} = \begin{bmatrix} -\frac{a_1(p)}{a_0(p)} & -\frac{a_2(p)}{a_0(p)} & -\frac{a_3(p)}{a_0(p)} & -\frac{a_4(p)}{a_0(p)} & -\frac{a_5(p)}{a_0(p)} \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (34)$$

$$\begin{aligned}
aa_{11} &= -432.45 - 2.3529*10^{-4} p_3 \\
aa_{12} &= -2.1733*10^5 - 2.3529*10^{-4} p_1 - 0.0137 p_2 - 0.0561 p_3 \\
aa_{13} &= -0.0561 p_1 - 12.779 p_2 - 1.8182 p_3 - 5.3476*10^{-7} p_2 p_3 - \\
&\quad -1.9455*10^7 \\
aa_{14} &= -4.7455*10^8 - 1.8182 p_1 - 1144.0 p_2 - 5.3476*10^{-7} p_1 p_2 - \\
&\quad -1.0695*10^{-4} p_2 p_3 \\
aa_{15} &= -1.0695*10^{-4} p_1 p_2 - 2.7903*10^4 p_2
\end{aligned}$$

We substitute the central vector of parameters

$$p^c = \begin{bmatrix} 14000 & 10000 & 480 \end{bmatrix}^T \quad (35)$$

in (43) and based on (42) determine the maximum eigenvalue of A

$$\lambda_{\max}(p^c) = -0.5879 . \quad (36)$$

Next we apply the method described in Section 3 and get the outer bounds of the exact range of the maximum eigenvalue

$$\boldsymbol{\lambda}'_{\max} = \left[-0.77599, \quad -0.40000\right]. \quad (37)$$

The exact range is calculated by applying the method from Section 4

$$\boldsymbol{\lambda}^*_{\max} = \left[-0.7647, \quad -0.4118\right]. \quad (38)$$

6. CONCLUSION

The classical criteria of stability (Raus and Frazer-Duncan criteria) in interval form based on the interval extensions of the functions studied.

A method proposed in Section 3 obtains the outer bounds $\boldsymbol{\lambda}'_{\max}$. reduces to solving the incomplete quadratic system (22). The method is applicable if the solution r of system (22) is positive.

A version of this method for finding the outer solution can be used for determining the outer bounds $\boldsymbol{D}_{il}$ of the derivatives $\frac{\partial y_i}{\partial p_l} = D_{il}$. If these bounds satisfied monotonicity conditions (30) the method, proposed in Section 4, can provide the exact solution for the eigenvalue range of the maximum eigenvalue.

A numerical example for analyzing the stability of calibration device for high accuracy acceleration transducers illustrates the applicability of the above methods to determine the outer bounds and the exact range of the maximum eigenvalue of the system (33).

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