Външни оценки на собствените стойности на интервални матрици – комплексен случай

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Резюме: Анализът на устойчивостта на линейни вериги и системи при интервална неопределеност на параметрите може да се сведе до оценяване на собствените числа на интервални матрици. В настоящата статия е разгледана задачата за определяне на външни граници на собствени стойности на матрици с интервални елементи – комплексен случай. Предложен е метод за решаването и. Той се състои в решаване на нелинейна система от 2n уравнения с 2n неизвестни. Последната е слабо нелинейна и численото и решение не създава трудности. Разгледан е числов пример за илюстриране приложимостта на метода.

Outer bounds on the eingenvalues of interval matrices - the complex eingenvalues case

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Abstract: Stability analysis of linear circuits and systems under interval parameters uncertainties can be equated to estimating the eigenvalues of interval matrices. In this paper, the problem of determining outer bounds on the ranges of the eigenvalues of interval matrices with complex eigenvalues, is considered. A method for computing such bounds is suggested. It consist of setting up and solving a system of 2n nonlinear equations with 2n unknowns. The latter system is only mildly nonlinear and its solution presents no numerical difficulties. An example illustrating the applicability of the method suggested is provided.

1. Introduction

The problem of estimating the range of the eigenvalues of matrices is the main problem in stability analysis of linear circuits and systems under interval uncertainties. There are a lot of methods for its solving for the real case (see [2] – [5]). Recently, a method was proposed for computing the outer bounds on the real eigenvalues of matrices with interval components which estimations are relatively smaller conservative than other methods [1].

In this paper, as a generalization of the approach from [1] a method for obtaining bounds on the eigenvalues in the complex case is suggested. Its results are compared to the results of the stochastic method “Monte Carlo” which gives inner bounds on the eigenvalues. Also, the method guaranties that the bounds are outer bounds, i.e. they contain the actual eigenvalue ranges. Furthermore, the method yields bounds that seem to be rather tight, i.e. with relatively small conservatism. From computational point of view, the method suggested reduces to solving a corresponding non-linear algebraic system than consists of 2n equations in 2n unknowns.

2. Problem statement

Let A be a real (n x n) matrix, $\boldsymbol{A}$ - an interval matrix containing A, and A^-, A^+, A_0 and R_A – the left end, the right end, the center and the radius of $\boldsymbol{A}$, respectively. We define the following "perturbed" eigenvalue problem:

$$A.x = \lambda.x,\ A\in\boldsymbol{A} = [A^-, A^+] = A_0 + [\ -R_A, R_A] \qquad (1)$$

In this paper, we are interested in the complex eingenvalues of (1). Let $\boldsymbol{\lambda}^*(A)$ denote such a complex eigenvalue while $x^{(k)}(A) = (x_1^{(k)}(A), x_2^{(k)}(A), \dots , x_n^{(k)}(A))$ be the corresponding (complex) eigenvector, $k = 1, \dots , n', n' \leq n$. We need the following assumption:

Assumption A_1: For all possible variations of $A\in\boldsymbol{A}$, the eigenvalue problem remains structurally stable i.e. for any $k\in K = 1, \dots , n'$, all $\lambda^{(k)}(A)$ *and* $x^{(k)}(A)$, corresponding to all $A\in\boldsymbol{A}$, remain complex, while the remaining (n – n') eigenvalues remain real.

The real eigenvalues which are the last (n – n') eigenvalues can be determined by the algorithm proposed in [1].

We proceed to considering the complex eigenvalue case. For simplicity, we shall henceforth drop the index k indicating the number of the eigenvalue. On account of Assumption A_1, the ranges

$$\boldsymbol{\lambda}^*_{Re} = \{\ \lambda_{Re}(A): A\in\boldsymbol{A}\} \qquad (2a)$$

$$\boldsymbol{\lambda}^*_{Im} = \{\ \lambda_{Im}(A): A\in\boldsymbol{A}\} \qquad (2b)$$

are real intervals. In checking the dynamic stability of linear systems, the ideal would be to determine $\boldsymbol{\lambda}^*_{Re}$ and $\boldsymbol{\lambda}^*_{Im}$. Since presently this seems to be an intractable problem, we usually settle for an outer approximations λ_{Re} and λ_{Im} of $\boldsymbol{\lambda}^*_{Re}$ and $\boldsymbol{\lambda}^*_{Im}$, i.e. λ_{Re} and λ_{Im} must include $\boldsymbol{\lambda}^*_{Re}$ and $\boldsymbol{\lambda}^*_{Im}$:

$$\boldsymbol{\lambda}^*_{Re} \subset \boldsymbol{\lambda} \qquad (3a)$$

$$\boldsymbol{\lambda}^*_{Im} \subset \boldsymbol{\lambda} \qquad (3b)$$

Thus, the problem at hand is the following:

Problem P_1: Find an outer bound λ_{Re} and λ_{Im} on $\boldsymbol{\lambda}^*_{Re}$ and $\boldsymbol{\lambda}^*_{Im}$, i.e. an estimation of $\boldsymbol{\lambda}^*_{Re}$ and $\boldsymbol{\lambda}^*_{Im}$ having the inclusion property (3a) and (3b).

In this paper, we suggest a method for finding a "good" outer bound λ_{Re} and λ_{Im} on $\boldsymbol{\lambda}^*_{Re}$ and $\boldsymbol{\lambda}^*_{Im}$, that is a bound with a small overestimation.

To simplify presentation of the method (without any loss of generality), we need a second assumption. Let the pair (x^0, λ^0) be the solution of the (center, nominal) problem

$$A_0.x = \lambda.x \qquad (4)$$

Assumption A_2: We assume that the absolute value of the n-th component $\left|x_n^0\right|$ of x^0 is the largest component of the other components, i.e.

$$\left|x_n^0\right| \geq \left|x_i^0\right|,\ i \neq n \quad (5)$$

If p-th component is the largest component, we need to interchange the places of the p-th and n-th row in A matrix as well as the position of the components x_p and x_n.

Now x^0 is "normalized" by letting

$$\left|x_{nRe}^0\right| = 1 \quad (6a)$$

$$x_{nIm}^0 = \frac{x_{nIm}^0}{\left|x_{nRe}^0\right|} = \alpha_{2n} \quad (6b)$$

Further, we require that (6a) and (6b) be also valid for

$$\left|x_{nRe}(A)\right| = 1, \qquad \forall A \in \boldsymbol{A} \quad (6c)$$

$$\left|x_{nIm}(A)\right| = \alpha_{2n}, \qquad \forall A \in \boldsymbol{A} \quad (6d)$$

Conditions (6c) and (6d) simplify the method which will be presented in the next section.

3. Problem solution

We introduce the 2n-dimensional real vector

$$Y = (y_1, y_2, \dots, y_{2n}) \quad (7)$$

with

$$y_i = x_{iRe}(A),\ i = 1, \dots, (n-1) \quad (7a)$$

$$y_n = \lambda_{Re}(A), \quad (7b)$$

$$y_i = x_{iIm}(A),\ i = (n+1), \dots, (2n-1) \quad (7c)$$

$$y_{2n} = \lambda_{Im}(A), \quad (7d)$$

Using (7) and (6), (1) is rewritten as

$$\left|\begin{array}{l}
a_{11}\cdot y_1 + a_{12}\cdot y_2 + \dots + a_{1,(n-1)}\cdot y_{n-1} - y_n\cdot y_1 + a_{1n} + y_{2n}\cdot y_{n+1} = 0 \\
a_{21}\cdot y_1 + a_{22}\cdot y_2 + \dots + a_{2,(n-1)}\cdot y_{n-1} - y_n\cdot y_2 + a_{2n} + y_{2n}\cdot y_{n+2} = 0 \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
a_{n1}\cdot y_1 + a_{n2}\cdot y_2 + \dots + a_{n,(n-1)}\cdot y_{n-1} - y_n\cdot 1 + a_{nn} + y_{2n}\cdot\alpha_{2n} = 0 \\
a_{11}\cdot y_{n+1} + a_{12}\cdot y_{n+2} + \dots + a_{1,(n-1)}\cdot y_{2n-1} - y_n\cdot y_{n+1} + a_{1n} - y_{2n}\cdot y_1 = 0 \\
a_{21}\cdot y_{n+1} + a_{22}\cdot y_{n+2} + \dots + a_{2,(n-1)}\cdot y_{2n-1} - y_n\cdot y_{n+2} + a_{2n} - y_{2n}\cdot y_2 = 0 \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
a_{n1}\cdot y_{n+1} + a_{n2}\cdot y_{n+2} + \dots + a_{n,(n-1)}\cdot y_{2n-1} - y_n\cdot\alpha_{2n} + a_{nn} - y_{2n}\cdot 1 = 0
\end{array}\right. \quad (8a)$$

where

$$\boldsymbol{A_{ij}} = [A_{ij}^-, A_{ij}^+] \quad (8b)$$

$$Y_{Re} = [y_1, y_2, \dots, y_n]^T \quad (8c)$$

$$Y_{Im} = [y_{(n+1)}, y_{(n+2)}, \dots, y_{2n}]^T \quad (8c)$$

$$Y' = diag\{y_n, y_n, \dots, y_n, 1\} \quad (8d)$$

$$Y'' = diag\{y_{2n}, y_{2n}, \dots, y_{2n}, \alpha_{2n}\} \quad (8e)$$

System (8a) is a non-linear (more precisely, a quadratic) interval system because of the products $y_n.y_i$ (i = 1, 2, …, (n-1)) in the first (n-1) equations in (8a) and products $y_{2n}.y_{(n+i)}$ (i = 1, 2, …, (n-1)) in next (n+1) - (2n-1) equations in (8a).

Let $\boldsymbol{y}_i^*$ denote the range of the i-th component $y_i\ (A),\ A \in \boldsymbol{A}$**,** of the solution to (8a). Let $\boldsymbol{Y}^*$ be the vector made up of y_i^*. Consider the following problem:

Problem P_2: Find an outer solution $\boldsymbol{Y}$ to (8), i.e. a solution enclosing the range vector $\boldsymbol{Y}^*$*:*

$$\boldsymbol{Y}^* \subset \boldsymbol{Y} \qquad (9)$$

Obviously, the n-th and 2n-th components of the solution $\boldsymbol{Y}$ to Problem P_2 is a solution to the original Problem P_1.

We now proceed to solving Problem P_2. The approach adopted is based on an idea suggested recently in [6]. If $z = z_0 + u \in Z$ and $t = t_0 + v \in T$, with Z and T being intervals whose centers are z_0 and t_0, respectively, then

$$u.v \in t_0.u + z_0.v + [\,-R_u.R_v,\ R_u.R_v] \qquad (10)$$

where R_u and R_v are the respective radiuses. We apply (10) to express the products in (8a) after letting

$$a_{ij} = a_{ij}^0 + u_{ij},\ y_i = y_i^0 + v_i \qquad (11)$$

Having in mind that the centers a_{ij}^0 and y_i^0 satisfy system (8a) and following the techniques of [6] we get the system:

$$\left|\begin{array}{l}
(a_{11}^0 - y_n^0).v_1 + a_{12}^0.v_2 + \ldots + a_{1,(n-1)}^0.v_{n-1} - y_1^0.v_n + y_{(n+1)}^0.v_{2n} = B_1 \\
a_{21}^0.v_1 + (a_{22}^0 - y_n^0).v_2 + \ldots + a_{2,(n-1)}^0.v_{n-1} - y_2^0.v_n + y_{(n+2)}^0.v_{2n} = B_2 \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
a_{(n-1),1}^0.v_1 + a_{(n-1),2}^0.v_2 + \ldots + (a_{(n-1),(n-1)}^0 - y_n^0).v_{n-1} - y_{(n-1)}^0.v_n + y_{(2n-1)}^0.v_{2n} = B_{n-1} \\
a_{n1}^0.v_1 + a_{n2}^0.v_2 + \ldots + a_{n,(n-1)}^0.v_{n-1} - 1.v_n + \alpha_{2n}.v_{2n} = B_n \\
(a_{11}^0 - y_n^0).v_{(n+1)} + a_{12}^0.v_{(n+2)} + \ldots + a_{1,(n-1)}^0.v_{2n-1} - y_{(n+1)}^0.v_n - y_1^0.v_{2n} = B_{(n+1)} \\
a_{21}^0.v_{(n+1)} + (a_{22}^0 - y_n^0).v_{(n+2)} + \ldots + a_{2,(n-1)}^0.v_{(2n-1)} - y_{(n+2)}^0.v_n - y_2^0.v_{2n} = B_{(n+2)} \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
a_{(n-1),1}^0.v_{(n+1)} + a_{(n-1),2}^0.v_{(n+2)} + \ldots + (a_{(n-1),(n-1)}^0 - y_n^0).v_{(2n-1)} - y_{(2n-1)}^0.v_n - y_{(2n-1)}^0.v_{2n} = B_{(2n-1)} \\
a_{n1}^0.v_1 + a_{n2}^0.v_2 + \ldots + a_{n,(n-1)}^0.v_{n-1} - \alpha_{2n}v_n - 1.v_{2n} = B_{2n}
\end{array}\right. \qquad (12a)$$

It can be easily checked that the radius of B_i is

$$R(B_i) = \sum_{j=1}^{n-1} R(A_{ij}).\left|y_j^0\right| + \sum_{j=1}^{n-1} R(A_{ij}).R(Y_j) + R(A_{in}) + [R(Y_n).R(Y_i)] + [R(Y_{2n}).R(Y_{n+i})], \quad i = 1..(n-1)$$

(12b)

$$R(B_n) = \sum_{j=1}^{n} \left|y_j^0\right|.R(A_{nj}) + \sum_{j=1}^{n-1} R(A_{nj}).R(Y_j) + R(A_{nn}) \qquad (12c)$$

$$R(B_{n+i}) = \sum_{j=1}^{n-1} R(A_{ij}).\left|y_{n+j}^0\right| + \sum_{j=1}^{n-1} R(A_{ij}).R(Y_{n+j}) + R(A_{in}) + [R(Y_n).R(Y_{n+i})] + [R(Y_{2n}).R(Y_i)], \quad i = 1..(n-1)$$

(12d)

$$R(B_{2n}) = \sum_{j=1}^{n} \left| y_{n+j}^{0} \right| . R(A_{nj}) + \sum_{j=1}^{n-1} R(A_{nj}).R(Y_{n+j}) + R(A_{nn}) \qquad (12e)$$

where $R(A_{ij})$ is the radius if u_{ij} (equivalently of a_{ij}) and $R(A_i)$ is the radius of v_j (equivalently of y_j). Now system (12) can be written in a compact form

$$\tilde{A}_0 . v = B \qquad (13)$$

where $\tilde{A}_0$ is the real coefficient matrix in (12a). Let $C = \left| \tilde{A}_0^{-1} \right|$ and $R(Y) = (R(Y_1), R(Y_2), \ldots, R(Y_{2n})) = r = (r_1, r_2, \ldots, r_{2n})$. From (13)

$$r = C.R(B) \qquad (14)$$

Now we introduce the following matrices:

1. $\tilde{R}$ is a *(2n x 2n)* block matrix in the form:

$$\tilde{R} = \begin{bmatrix} \{R_{ij}\} & Z \\ Z & \{R_{ij}\} \end{bmatrix}$$

where $R_{ij} = [R(\mathbf{A_{ij}})]$, i = 1..n, j = 1..n; Z = [z_{ij}] in (n x n) matrix with zero elements;

2. $\breve{R}$ is *(2n x 2n)* block matrix as:

$$\breve{R} = \begin{bmatrix} R' & Z \\ Z & R' \end{bmatrix}$$

where R' is the same as R_{ij} expect for the last column whose elements are now zero;

$$\left| Y^0 \right| = \begin{bmatrix} \left| Y^0_{\mathrm{Re}} \right| \\ \left| Y^0_{\mathrm{Im}} \right| \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \left| y^0_1 \right| & \left| y^0_2 \right| & \ldots & \left| y^0_n \right| \end{bmatrix}^T \\ \begin{bmatrix} \left| y^0_{n+1} \right| & \left| y^0_{n+2} \right| & \ldots & \left| y^0_{2n} \right| \end{bmatrix}^T \end{bmatrix}$$

3. Y^0 is 2n-th vector column:

4. $\breve{R}^{n}$ and $\breve{R}^{2n}$ are diagonal (n x n) matrixes with $R(Y_n)$ and $R(Y_{2n})$ in the main diagonal respectively.

5. r_{Re}' and r_{Im}' are the same as vectors r_{Re} and r_{Im} expect for the n-th element of r_{Re}' which is 1 and n-th element of r_{Im}' which is α_{2n}.

6. $R^{is"}$ and $R^{ss"}$ are n-th vector column in the form:

$$R^{in"} = [R(A_{1n}) \quad R(A_{2n}) \quad \ldots \quad R(A_{(n-1)n}) \quad 0]^T$$

$$R^{nn"} = [0 \quad 0 \quad \ldots \quad 0 \quad R(A_{nn})]^T$$

$$r = R(Y) = \begin{bmatrix} r_{Re} \\ r_{Im} \end{bmatrix} = C.\tilde{R}.\begin{bmatrix} \left| Y^0_{Re} \right| \\ \left| Y^0_{Im} \right| \end{bmatrix} + C.\breve{R}.\begin{bmatrix} r_{Re} \\ r_{Im} \end{bmatrix} + C.\begin{bmatrix} \breve{R}^{n}.r_{Re}' \\ \breve{R}^{n}.r_{Im}' \end{bmatrix} + C.\begin{bmatrix} \breve{R}^{2n}.r_{Im}' \\ \breve{R}^{2n}.r_{Re}' \end{bmatrix} + C\begin{bmatrix} R^{in"} \\ R^{in"} \end{bmatrix} + C\begin{bmatrix} R^{nn"} \\ R^{nn"} \end{bmatrix}$$

Using (12b), (12c), (12d), (12e) and the new notations, (14) can be written in the form:

$$r = d + D.r + C.\begin{bmatrix} \breve{R}_{n}.r_{Re}' \\ \breve{R}_{n}.r_{Im}' \end{bmatrix} + C.\begin{bmatrix} \breve{R}_{2n}.r_{Im}' \\ \breve{R}_{2n}.r_{Re}' \end{bmatrix} \qquad (15)$$

with (16a)

$$d = C.\tilde{R}.\left|Y^0\right| + C\begin{bmatrix} R_{in}'' \\ R_{in}'' \end{bmatrix} + C\begin{bmatrix} R_{nn}'' \\ R_{nn}'' \end{bmatrix}, \qquad D = C.\breve{R} \tag{16b}$$

The matrix equation (16) is a non-linear real value (non-interval) system of 2n equations of 2n unknowns r_i. The solution of (16a) for positive r_i solves Problem P_2. Indeed

$$\boldsymbol{y_i} = y_i^0 + [- r_i, r_i],\ i = 1, 2, \dots, 2n. \tag{17}$$

Following [7] it can be shown that

$$\boldsymbol{y_i}^* \subset \boldsymbol{y_i},\ i = 1, 2, \dots, 2n, \tag{18}$$

i.e. the intervals (17) are really outer bounds on the ranges y_i^* for all i. Hence, the interval

$$\boldsymbol{y_n} = y_n^0 + [- r_n, r_n] \tag{19a}$$

$$\boldsymbol{y_{2n}} = y_{2n}^0 + [- r_{2n}, r_{2n}] \tag{19b}$$

is the solution to the original problem P_1 and is, in fact, a bound $\boldsymbol{\lambda}$ on λ^* satisfying the inclusion (3a) and (3b).

Thus, it has been shown that the original problem P_1 reduces to solving the non-linear (incomplete quadratic) system (16a). Since R_{ij} are, most often, percents of a_{ij}^0 and (16a) is only mildly non-linear, it's solution does not present any problem.

4. Numerical example

The applicability of the method will be illustrated by the example with n = 2:

$$\left| \begin{array}{l} a_{11}.x_1 + a_{12}.x_2 - \lambda.x_1 = 0 \\ a_{21}.x_1 + a_{22}.x_2 - \lambda.x_2 = 0 \end{array} \right. \tag{20}$$

Here

$$\left| \begin{array}{ll} a_{11}^0 = -3.2 & a_{12}^0 = 2 \\ a_{21}^0 = -4 & a_{22}^0 = 1.8 \end{array} \right. \tag{20a}$$

and $R_{ij}\ (A)$= 0.052, $i = 1, 2;\ j = 1, 2$.

First, we determine the centre of the eigenvalues

$$\Lambda^o = [- 0.7 - j1.322876, - 0.7 + j1.322876]^T \tag{21}$$

and

$$\lambda_{maxRe} = \max_i\{\lambda^{(i)}{}_{Re}(A_0)\} = -0.7 \tag{22}$$

For this example, the index k corresponding to λ_{max} is k=1 and the corresponding normalized vector of the centres of the variables from (5) and (6) is:

$$X^0 = [x_{1Re}^{0'}, 1, x_{1Im}^{0'}, x_{2Im}^{0'}]^T = [x_{1Re}^0/ x_{2Re}^0,\ 1,\ x_{1Im}^0/ x_{2Re}^0,\ x_{2Im}^0/ x_{2Re}^0]^T \tag{24}$$

According to (7) vector Y is:

$$Y = [y_1, y_2, y_3, y_4]^T = [x_{1Re}^{'}, \Lambda_{Re}, x_{1Im}^{0'}, \Lambda_{Im}]^T \tag{25}$$

so

$$Y^0 = [y_1^0, y_2^0, y_3^0, y_4^0]^T = [0.625, -0.7, 0.3307, 1.3307]^T \quad (26)$$

So the system (8a) is:

$$\left| \begin{array}{l} a_{11}.y_1 + a_{12} - y_2.y_1 + y_4.y_3 = 0 \\ a_{21}.y_1 + a_{22} - y_2.1 + y_4.\alpha_4 = 0 \\ a_{11}.y_3 + a_{12} - y_2.y_3 - y_4.y_1 = 0 \\ a_{21}.y_3 + a_{22} - y_2.\alpha_4 - y_4.1 = 0 \end{array} \right. \qquad \alpha_4 = x_{2Im}^{0'} = 0 \quad (27)$$

Applying (10) and simplify the system get the following non-linear system:

$$\left| \begin{array}{l} 0.9575\ r_1 + 0.2088\ r_3 + 0.75597\ r_2 r_3 + 0.75597\ r_1 r_4 + 0.3469 = 0 \\ -r_2 + 0.835363\ r_3 + 3.024\ r_2 r_3 + 3.024\ r_1 r_4 + 1.1115 = 0 \\ 0.2088\ r_1 + 0.9575\ r_3 + 0.75597\ r_1 r_2 + 0.75597\ r_3 r_4 + 0.3959 = 0 \\ 0.835363\ r_1 - r_4 + 3.024\ r_1 r_2 + 3.024\ r_3 r_4 + 1.357 = 0 \end{array} \right. \quad (28)$$

The solution of (28) is:

$$R(Y) = [R(y_1), R(y_2), R(y_3), R(y_4)]^T = [0.1053, 0.268341, 0.0556, 0.92343]^T \quad (29)$$

Finally, from (21) and (29):

$$\Lambda_{Re} = [-0.7 - 0.2685445, -0.7 + 0.26885445] = [-0.9685445, -0.4314555] \quad (30)$$

Thus, in view of (30), we conclude that the estimations of the real part of the complex eigenvalues of interval matrix A at R(A) = 0.17 with our approach are outer bounds. We can demonstrate that with compare to results of "Monte Carlo" method, which gives inter bounds of the estimation of the real part of the eigenvalues: λ_{Re} = [-0.868759, - 0.532679].

5. Conclusion

The problem of bounding the complex eigenvalues of interval matrices has been considered. It is related to the problem of assessing the robust stability of linear circuits or systems having interval parameters. A method for determining outer bounds on the eigenvalue ranges has been suggested. It requires the evaluation of the complex eigenvalues and the corresponding complex eigenvectors from (4) for the center (nominal parameters) matrix A_0. The method essentially consists of setting up and solving the system of 2n non-linear equations (16a) for the positive solutions r_i, i = 1, 2, … , 2n. The solution of the original problem is then found by the n-th and 2n-th radii r_n and r_{2n} according to formulae (19a) and (19b).

The approach herein suggested is a generalization of the case of real eigenvalues (see [1]). The form of the non-linear systems (16) for determining the complex eigenvalues is the same except that the number of components in (16a) is twice as much as it in the real case because there is a real and imaginary part of the eigenvalues and eigenvectors and the size of the system (16a) increases to 2n.

A further possible generalization is to encompass matrices whose elements are non-linear functions of a certain number of interval parameters.

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