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A Linear Programming Implementation of a Interval Method for Global Non-Linear DC Analysis

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Abstract

A modification of the previous author method for finding the sett of all operating points of non-linear resistive circuits is suggested. The original method is based on an approximation of every single variable function (circuit equations are in a hybrid representation form) by an appropriate linear interval function, i.e. by a real linear function having an additive interval constant. The improved approach uses linear programming technique to update the current interval "box" instead of the originally used interval hull of the solution set of the linearized interval system.

Numerical experiments show that the version suggested reduces almost double the number of the iterations in comparison with the original method for the examples considered.

1. Introduction

The following problem of global analysis of nonlinear resistive circuits is considered. Let $\mathbf{f}:\mathbf{X}^{(0)} \subset \mathbf{R}^n \longrightarrow \mathbf{R}^n$ be the function describing the dc operation of the circuit studied where n is the number of equations and $\mathbf{X}^{(0)}$ is the interval "box" in $\mathbf{R}^n$. Find, with certainty, the set $S(f, X^{(0)}) = \{x^{(1)}, x^{(2)}, \ldots, x^{(p)}\}$ of all real solutions $x^{(s)}, s = 1,2,\ldots, p$ of the system

$$\mathbf{f(x)} = 0 \quad (1)$$

$$\mathbf{x} \in \mathbf{X}^{(0)} = (X_1^{(0)}, X_2^{(0)}, \ldots, X_n^{(0)}) \quad (2)$$

where

$$X_i^{(0)} = \left[\underline{x_i^{(0)}}, \overline{x_i^{(0)}}\right], \; i = 1,2,\ldots,n \quad (3)$$

and each component f_i of f is in the so-called separable form

$$f_i = \sum_{i=1}^{n} f_{ij}(x_j) \quad (4)$$

As it is stressed in [1], the present problem statement is more general as compared with other publications (e.g. [2]-[10]) in the following aspects:

i) unlike [2], [4]-[10] where special cases of (4) are treated (hybrid representation form with $f_{ij}(x_j)=0,\ j\neq i$, or a special description [3] of circuits containing bipolar and MOS transistors with $f_{ij}(x_j)\neq 0$ only for $j=i$ and $j=i+1$ or $j=i$ and $j=i-1$) now the full separable description (2) is considered;

ii) the class of functions $f_{ij}(x_j)$ is more general whereas all the previous papers (cf. [2]-[10] and their references) admit only one function (most often $f_{ii}(x_i)$) of a special type.

In this paper, an improvement of the interval method from [1] is suggested which is capable of handling the general problem formulated above. The improvement is based on application of linear programming techniques.

The paper is outlined as follows. In the Section 2 we give a background of the method of [1] and present the new ideas for improving the efficiency. In Section 3 we give two examples and compare the results with the original method [1] and we will end up with some conclusions in Section 4.

2. The improved method

Let $\mathbf{X} = (X_1, X_2, \ldots, X_n)$ be the "subbox" processed at the current iteration. Unlike previous interval methods where interval derivatives [4], [5] or interval slopes [6], [7] are used, in [1] each $f_{ij}(x_j)$ is approximated in X_j by the following linear function

$$L_{ij}(X_j) = B_{ij} + a_{ij}x_j, \; x_j \in X_j \quad (5)$$

where $B_{ij} = \left[\underline{b_{ij}}, \overline{b_{ij}}\right]$ is an interval while a_{ij} is a real number. Both Bij and a_{ij} are determined such that the following inclusion property holds [1]

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$$f_{ij}(x_j) \in B_{ij} + a_{ij}x_j,\ x_j \in X_j \quad (6)$$

A simple procedure for finding $a_{ij}, \underline{b}_{ij}, \overline{b}_{ij}$ for the case of continuously differentiable functions (see Fig. 1) is suggested in [1]. The procedure is motivated by elementary geometrical considerations (see Fig. 1) and can be readily modified for the case of only continuous (in particular piecewise-linear or even discontinuous functions.

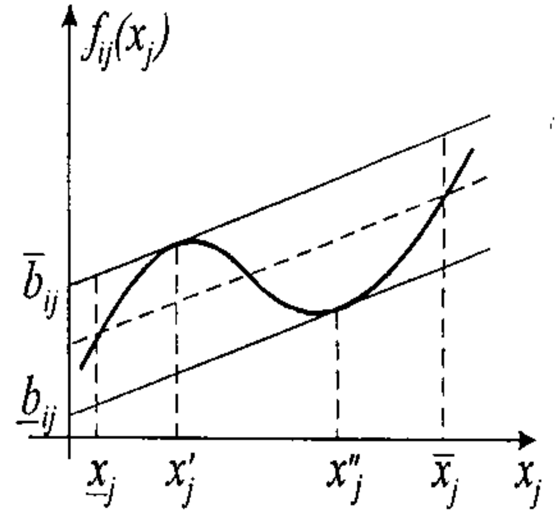

Fig. 1. Geometrical illustration of the linear interval approximation of $f_{ij}(x_j)$ in $X_{ji} = \left[\underline{x}_j, \overline{x}_j\right]$.

After the parameters $a_{ij}, \underline{b}_{ij}, \overline{b}_{ij}$ $i, j = 1,2,\dots,n$ have been determined a real matrix

$$\mathbf{A} = \left\{-a_{ij}\right\} \quad (7)$$

and an interval vector $\mathbf{B} = (B_1, B_2, \dots, B_n)$ are formed with

$$B_i = \sum_{j=1}^{n} B_{ij} = \left[\underline{b}_i, \overline{b}_i\right] \quad (8)$$

On account of (4), (6), and (8)

$$f_i(x) \in \sum_{j=1}^{n} a_{ij}x_j + B_i,\ i = 1,2,\dots,n,\ \mathbf{x} \in \mathbf{X} \quad (9)$$

or in vector form

$$\mathbf{f(x)} \in -\mathbf{Ax} + \mathbf{B},\ \mathbf{x} \in \mathbf{X} \quad (10)$$

If **y** is a solution of (1) in **X**, then **f(y)=0** and by (10) $0 \in -\mathbf{Ay} + \mathbf{B}$, $\mathbf{y} \in \mathbf{X}$. Based on the above relations the following theorems are stated in [1].

Theorem 1. *All the solutions to system (1) contained in* **X** *are also contained in the solution set of the system*

$$-\mathbf{Ax} + \mathbf{B} = 0,\ \mathbf{b} \in \mathbf{B} \quad (11)$$

where b is any real vector contained in **B**.

Theorem 2. *All the solutions* **y** *to (1) in* **X** *are also contained in the intersection*

$$\mathbf{P(X)} = \mathbf{S(X)} \cap \mathbf{X} \quad (12)$$

Let **H(S,X)** denote the interval hull of **S(X)**. Then it follows from (11) that

$$\mathbf{Y} = \mathbf{H(S,X)} = \mathbf{A}^{-1}\mathbf{B} \quad (13)$$

and the iterative procedure for reduction the current interval "box" $\mathbf{X}^{(k)}$ is

$$\mathbf{X}^{(k+1)} = \mathbf{Y}^{(k)} \cap \mathbf{X}^{(k)},\ k \geq 0 \quad (14)$$

where

$$\mathbf{Y}^{(k)} = (\mathbf{A}^{(k)})^{-1}\mathbf{B}^{(k)} \quad (15)$$

and the real matrix $\mathbf{A}^{(k)}$) and the interval vector $\mathbf{B}^{(k)}$ correspond to the current interval "box" $\mathbf{X}^{(k)}$.

Theorem 3 (Existence). *Let* $\mathbf{f}: \mathbf{D} \subset \mathbf{R}^n \longrightarrow \mathbf{R}^n$ *be a continuously differentiable function in the domain* **D** *and* $\mathbf{X}^{(0)} \subset \mathbf{D}$. *Introduce the interval operator*

$$\mathbf{K}(\mathbf{X}^{(k)}) = \mathbf{H}(\mathbf{S}^{(k)}, \mathbf{X}^{(k)}) = -(\mathbf{A}^{(k)})^{-1}\mathbf{B}^{(k)},\ k \geq 0 \quad (16)$$

where $\mathbf{A}^{(k)}$) *and* $\mathbf{B}^{(k)}$, *correspond to the current box* $\mathbf{X}^{(k)}$. *Then, if at some k-th iteration*

$$\mathbf{K}(\mathbf{X}^{(k)}) \subseteq \mathbf{X}^{(k)} \quad (17)$$

the above inclusion implies the existence of a solution to (1) in $\mathbf{X}^{(k)}$.

Theorem 4 *(Convergence). Suppose that*

$$\mathbf{K}(\mathbf{X}^{(k)}) \subset \mathbf{X}^{(k)} \quad (18)$$

holds for all $k \geq k_0$ *i.e. that the iterative procedure (14) converges to a solution* $\mathbf{x}^*$ *and the Jacobian* $\mathbf{J}(\mathbf{x}^*)$ *at the solution is not singular. Then the convergence rate towards* $\mathbf{x}^*$ *is superlinear.*

Theorem 5 *(Uniqueness). If at some iteration* k_0 *of the iterative procedure (14), the condition*

$$\mathbf{K}(\mathbf{X}^{(k_0)}) \subset \mathbf{X}^{(k_0)} \quad (19)$$

is satisfied and the real functions $f_{ij}(x_j)$ *are all strictly monotone in* $X_{ij}^{k_0}$, *then:*

a) there is a unique solution $\mathbf{x}^*$ *to (1) in* $\mathbf{X}^{(k_0)}$;

b) the modified procedure

$$\mathbf{X}^{(k+1)} = \mathbf{K}(\mathbf{X}^{(k)}) \cap \mathbf{C}^{(k_0)}\mathbf{B}^{(k)},\ k \geq k_0 \quad (20)$$

where $\mathbf{C}^{(k_0)} = (\mathbf{A}^{(k_0)})^{-1}$ *is a constant matrix while* $\mathbf{B}^{(k)}$ *is computed as in Procedure 1 (given in [1]), converges to* $\mathbf{x}^*$ *with superlinear rate.*

The aim of our considerations is to find out possibilities to reduce the current subbox $\mathbf{X}^{(k)}$ more than using the interval procedure (14), (15). Based on the special form of (10) (**A** is a real matrix and **B** is an interval vector), here we suggest to determine the lower and the upper bounds $\underline{x}_i^{(k+1)}$, $\overline{x}_i^{(k+1)}$ of the i-th component of $\mathbf{X}^{(k+1)}$ through linear programming technique, namely

$$\begin{aligned} \underline{x_i^{(k+1)}} &= \min(x_i) \\ -\mathbf{A}^{(k)}\mathbf{x}+\mathbf{b} &= \mathbf{0} \\ \mathbf{b} &\in \mathbf{B}^{(k)} \\ \mathbf{x} &\in \mathbf{X}^{(k)} \end{aligned} \qquad (21)$$

and

$$\begin{aligned} \overline{x_i^{(k+1)}} &= \max(x_i) \\ -\mathbf{A}^{(k)}\mathbf{x}+\mathbf{b} &= \mathbf{0} \\ \mathbf{b} &\in \mathbf{B}^{(k)} \\ \mathbf{x} &\in \mathbf{X}^{(k)} \end{aligned} \qquad (22)$$

In detailed form (21) and (22) become

$$\begin{aligned} \underline{x_i^{(k+1)}} &= \min(x_i) \\ -\mathbf{A}^{(k)}\mathbf{x} &\le -\underline{\mathbf{b}^{(k)}} \\ -\mathbf{A}^{(k)}\mathbf{x} &\ge -\overline{\mathbf{b}^{(k)}} \\ \mathbf{x} &\in \mathbf{X}^{(k)} \end{aligned} \qquad (23)$$

and

$$\begin{aligned} \overline{x_i^{(k+1)}} &= \max(x_i) \\ -\mathbf{A}^{(k)}\mathbf{x} &\le -\underline{\mathbf{b}^{(k)}} \\ -\mathbf{A}^{(k)}\mathbf{x} &\ge -\overline{\mathbf{b}^{(k)}} \\ \mathbf{x} &\in \mathbf{X}^{(k)} \end{aligned} \qquad (24)$$

Hence, the application of the new approach differs from the original method [1] in using (21) and (22) for all $i=1,2,\dots,n$ instead of (14) and (15).

Based on the these considerations, and Theorem 2 the following corollaries can be proved.

C o r o l l a r y 1. *All the solutions to system (1) contained in $\mathbf{X}^{(k)}$ are also contained in the solution set of (21) and (22) for $i=1,2,\dots,n$.*

C o r o l l a r y 2. *At each iteration k the new interval vector $\mathbf{X}^{(k+1)}$ obtained from (21) and (22) for $i=1,2,\dots,n$ is always narrower than the same one obtained from (14), (15) of the original method [1].*

C o r o l l a r y 3. *If for any i, there is no feasible solution of (21), (22) then there is no solution y in the current "box" $\mathbf{X}^{(k)}$.*

Finally, it should be mentioned that using the suggested approach more calculations are necessary. The amount of calculations due to solving the linear programming problem usually is compensated with the reduced number of iterations ensuring the prescribed accuracy.

A slight improvement of the linear programming formulation (21), (22) is possible. It is concerned with introducing additional constraints on each x_i, connected with the lower bound $\underline{f_i}$ of $f_i(x_i)$ namely

$$a_{ii}x_i + b_i \ge \underline{f_i}\,,\; i=1,2,\dots,n$$

which makes the admissible region for each x_i more narrow. This improvement is expected to reduce the number of iterations needed to solve the non-linear problem considered.

3. Numerical examples

The numerical performance of the present method has been tested on several systems of equations of the hybrid representation form. To illustrate the almost double reduction of the number of iterations of the improved method needed to ensure the desired accuracy, two examples are considered.

Example 1

A circuit containing 10 tunnel diodes and studied in [7] and [10] has a description of the form

$$\mathbf{f}(\mathbf{x}) = \varphi(\mathbf{x}) - \mathbf{H}\mathbf{x} - \mathbf{s} = \mathbf{0}$$

with

$$\varphi_i(x_i) = 2.5x_i^3 - 10.5x_i^2 + 11.8x_i\,,\; i=1,2,\dots,10$$

$$\mathbf{H}=\{h_{ij}\},\; h_{ij}=-1,\; i,j=1,2,\dots,10$$

$$\mathbf{s}=(-1\ -2\ \dots\ -10)^T$$

The initial box $\mathbf{X}^{(0)}$ is defined by

$$x_i\in[-1,4],\; i=1,2,\dots,10$$

and the accuracy ε has been chosen to be 10^{-4}.

Two interval methods were applied in [7] to solve the global analysis problem considered. The first method denoted here as M1 is based on the use of interval derivatives while the second method designated as M2 employs interval slopes. Both the methods from [1] denoted as M3 and the improved version presented in the paper and denoted as M4 have also solved the problem considered and have found within the same accuracy $\varepsilon = 10^{-4}$ all the 9 solutions contained in $\mathbf{X}^{(0)}$. The results reveal that the method from [1] and especially the present modification are vastly superior to the best interval methods known as regards number of iterations.

Method	M1	M2	M3	M4
N	524143	116522	167	96

Table 1. Number of iterations for different interval methods.

Example 2

As a second example we consider the well known circuit containing four transistors and studied in [4]-[7].

The hybrid representation is chosen in the form (cf. formulae (11) in [6]), where the functions $f_{ii}(x_i)$ are

$$f_{ii}(x_i) = \exp(40x_i) - 1, \; i = 1,2,3,4$$

The nine solutions in the initial "box" $X^{(0)}$

$$x_1 \in [-1.1, 0.4], \; x_2 \in [-5, 0.4], \; x_3 \in [-1.6, 0.4] \; x_4 \in [-4, 0.4]$$

are obtained using the method from [1] and the suggested modification within accuracy $\varepsilon = 10^{-3}$. The number of the necessary iterations for the method from [1] is N_i=136 while the necessary iterations for the modified version is N_i=86.

In spite of the increased central processor time the number of the iterations is reduces almost double. This is very important when parallel computations are used.

4. Conclusions

A modified version of an interval method for global dc analysis has been suggested. The original method is based on the iterative procedure (14), (15) making use of the new interval representation (6) of each part of the separable function $f(x), i=1,2,\ldots,n$. The modified version appeals to the use of linear programming techniques to update the current interval box.

Numerical examples using circuit equations in a hybrid representation form (the functions $f_i(x), i=1,2,\ldots,n$ are separable) show that the new version reduces the number of iterations needed to located all the operating points within the accuracy chosen. In spite of the increased central processor time the suggested approach gives an approach how to reduce the number of iterations which is of great importance in parallel computations.

Furthermore, the linear programming implementation of the method from [1] may be superior to the original version even in the case of classical sequential computation if the computational efforts needed to handle the non-linear functions $f_i(x_i)$ are comparable with the amount of computation associated with solving the linear programming problems.

References

[1] L. V. Kolev, "A General Interval Method for Global Non-linear DC Analysis", *ICCTD'97 European Conference on Circuit Theory & Design*, (1997)., Budapest, Hungary, pp. , Sept. 1997.

[2] L.O. Chua and P.M. Lin, *Computer Aided Analysis of Electronic Circuits: Algorithms and Computational Techniques*, Prentice-Hall, Englewood Cliffs, NJ, 1975.

[3] M.Tadeusiewicz and K. Glovienka, "An Algorithm for Finding All Operating Points of Electronic Circuits Containing Intricate Models of Transistors", *Proc. ECCTD'95 European Conference on Circuit Theory & Design*, 127-130 (1995).

[4] L. V. Kolev, *Interval Methods for Circuit Analysis*, World Scientific, Singapore, New Jersey, 1993.

[5] L. V. Kolev and V. M. Mladenov, "An interval method for global non-linear dc circuits analysis", *Int. j. cir. theor. appl.*, 22, 233-241 (1994).

[6] L. V. Kolev and V. M. Mladenov, "Use of Interval Slopes in Implementing an Interval Method for Global Non-linear DC Circuits Analysis", *Int. j. cir. theor. appl.*, 25, 37-42 (1997).

[7] V. M. Mladenov and L. V. Kolev, "Interval Methods for Solving Cellular Neural Networks (CNNs) Equations", *The Third Int. Conf. on Electronics, Circuits and Systems (ICECS'96)*, Rodos, Greece, 13-16 October, (1996).

[8] L. V. Kolev, "Separability Property of the Non-Linear Equations for Resistive Circuits", (in Russian), *Electricity*, 2, 70-73 (1986).

[9] K. Yamamura and K. Horiuchi, "A globally and Quadratically Convergent Algorithm for Solving Non-linear Resistive Networks", *IEEE Trans. Computer-Aided Design*, 9, 487-499 (1990).

[10] K. Yamamura, "Finding All Solutions of Piecewise-Linear Resistive Circuits Using Simple Sign Tests", *IEEE Trans. Circuits Syst. I*, 40, 546-551 (1993).

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