X. International Symposium on Theoretical Electrical Engineering

September 6 - 9, 1999

Magdeburg, Germany

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WORST-CASE TOLERANCE ANALYSIS OF NON-LINEAR CIRCUITS USING AN INTERVAL METHOD

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Abstract

In this paper, a new iterative interval method applicable to both dc and ac worst-case tolerance analysis of non-linear circuits is presented. Besides being more general, it differs from other known methods in the way the linearized system arising at each iteration is set up and solved (approximately). Another distinction is the fact that now the initial linear tolerance problem (starting the iterations) corresponds to a circuit whose non-linear resistor characteristics are linearized around the corresponding nominal dc operating point of the original non-linear circuit studied.

1 Introduction

Interval methods have proved a reliable tool for solving the worst-case tolerance analysis problem for linear circuits [1]-[6]. The known methods are based on the following two basic approaches: (i) determining the range of a non-linear function relating the output variable to the interval parameters [1]-[4,Ch.2] and (ii) solving a corresponding system of linear equations having interval coefficients [3], [4,Ch.3], [5], [6]. The latter approach is more general in comparison to the former one since it permits the simultaneous determination of all output variables tolerances one is interested in. An additional advantage is the possibility to find approximate solutions in a much easier way than determination of the exact worst-case tolerances.

While the linear circuit worst-case tolerance analysis problem has been largely covered in the literature (e.g. [1]-[6]), its counterpart for non-linear circuits has drawn little attention: the only paper which touches upon the nonlinear tolerance problem seems to be [6], Sect. IV. B. An iterative interval method is suggested there for worst-case tolerance analysis of dc non-linear circuits. At each iteration, an associate linear dc tolerance problem is solved approximately, using an appropriate interval method. The approximate solution thus obtained must contain the tolerance solution sought for all iterations. Therefore, the iterations are initialized by selecting an initial interval vector $X^{(0)}$ which must contain the tolerance solution vector sought. The selection of a good initial vector $X^{(0)}$ remains an open problem. Indeed, if $X^{(0)}$ is chosen too large the iteration process will take too long to converge; conversely, if $X^{(0)}$ is too small it might not contain the tolerance solution. In this paper, a new iterative interval method applicable to both dc and ac worst-case tolerance analysis of non-linear circuits will be presented. Besides being more general, it differs from the method of [6] in the way the linearized system arising at each iteration is set up and solved (approximately). Another distinction is the fact that unlike [6] now the initial linear tolerance problem (starting the iterations) corresponds to a circuit whose non-linear resistor characteristics are linearized around the corresponding nominal dc operating point of the original non-linear circuit studied.

2 Problem statement

To simplify presentation, only the dc tolerance problem is presented here for the special case where the circuit contains only independent current-controlled non-linear resistors. Furthermore, it is assumed that each function $v_\rho = f_\rho(i_\rho)$ describing the v-i characteristic of the ρ-th non-linear element is known exactly (a more general setting, when $v_\rho = f_\rho(i_\rho) + b, b \in B$ where B is a given interval, can be easily encompassed by the new method). Following [4]-p.118, the following system can be set up

$$f_\rho(i_\rho) + r_\rho i_\rho = u_\rho,\ \rho = 1,\dots,m \qquad (1a)$$

$$\sum_{j=1}^{m} \alpha_{kj} i_j = 0,\ k = 1,\dots,n' \qquad (1b)$$

for a circuit having $n'+1$ nodes, m branches with m linear resistors, m non-linear resistors, and m independent voltage sources u_ρ.

Without loss of generality, it is assumed that only r_ρ (but not u_ρ) have tolerances, i.e. $r_\rho \in R_\rho = [\underline{r_\rho}, \overline{r_\rho}]$. Finally, system (1) will be written in vector form as

$$\Psi(x) = f(x) + Ax - b = 0,\ x \in R^n \quad (2a)$$

$$A \in A^I. \qquad (2b)$$

$$f_i(x) = f_i(x_i). \tag{2c}$$

$$A = \{a_{ij}\}, A^I = \{A_{ij}\}, a_{ij} \in A_{ij} \tag{2d}$$

where A_{ij} are independent intervals. The tolerance problem is to find an interval vector X^* which contains (as tightly as possible) the solution set S of (2)

$$S = \{x: \Psi(x) = 0, a_{ij} \in A_{ij}\} \tag{3}$$

The above formulation can be extended to more general dc non-linear circuits as well as to considering various ac tolerance problems in non-linear circuits.

3 The new method

3.1 Algorithm of the method.

Once again, for brevity, only the dc tolerance problem will be covered here.

Let a_{ij}^c and A^c denote the nominal values of the respective quantities. Using some interval methods for dc non-linear analysis [4], [7], the corresponding nominal solution x^c can be found. Now each $f_i(x_i)$ is linearized around x_i^c as

$$f_i(x_i) = f_i(x_i^c) + c_i(x_i - x_i^c),\ c_i = \frac{\partial f_i(x_i^c)}{\partial x_i} \tag{4}$$

and the diagonal matrix C with non-zero entries c_i and vector d with elements $d_i = f_i(x_i^c) - c_i x_i^c$, $i = 1,2,\ldots,n$ are formed. At this stage, the following linear dc tolerance analysis problem is solved

$$Cx + Ax = b - d,\ A \in A^I \tag{5}$$

Let $X^{(0)}$ denote an approximate interval solution of (5) having the property to contain the exact interval solution X^* of (5). At this point, each of $f_i(x_i)$ is represented within the interval $X_i^{(0)}$ by the following linear interval inclusion [7]:

$$f_i(x) = \alpha_i^{(0)} x_i + B_i^{(0)},\ x_i \in X_i^{(0)} \tag{6}$$

where $\alpha_i^{(0)}$ is a real number (the slope of f_i within $X_i^{(0)}$) while $B_i^{(0)}$ is a corresponding interval. On introducing the diagonal matrix $C^{(0)}$ whose entries are $\alpha_i^{(0)}$ and the interval vector $B^{(0)}$, the following linear dc tolerance analysis problem is next set up and solved:

$$C^{(0)}x + Ax = b - b^*,\ A \in A^I,\ b^* \in B^{(0)} \tag{7}$$

Let the approximate solution of (7) be $X^{(1)}$. At this stage, we form the union

$$X^{(2)} = X^{(0)} \cup X^{(1)} \tag{7'}$$

and an iterative procedure (Procedure 1) starts from (6) and (7) using (7'). Let $X^{(2v+1)}$ be the solution of the corresponding problem (7) at the v-th iteration. The iterative process is stopped when $X^{(2v+1)} \subseteq X^{(2v)}$. Now a second procedure (Procedure 2) is started which aims at reducing the width of the last interval vector $X^{(2v)}$ obtained at the end of Procedure 1. Let $X^{(0)} = X^{(2v)}$. The present procedure has the same structure as Procedure 1. The only difference is that now the union operation in (7') is replaced with the intersection operation

$$X^{(2)} = X^{(0)} \cap X^{(1)} \tag{7''}$$

The process stops when $X^{(2v)} \subseteq X^{(2v+1)}$. The solution of the original non-linear dc tolerance problem is then given by the result of Procedure 2.

The method is easily generalized to the ac case. The only difference is that now (5) and (7) are in complex form.

3.2 Solving linear tolerance problems

Since the iterative method involves the repetitive solution of system (7), its over all efficiency depends strongly on how efficiently (7) is solved at each iteration. In [6], the corresponding linear tolerance problem is solved approximately using an improved version of Hansen's method [8]. The same method could be also used for solving (7). A simpler method is, however, suggested here.

System (7) will be rewritten in the form

$$(A^c + \Delta)(x^c + u) = b^c + \delta,\ \Delta \in \Delta^I, \delta \in \delta^I \tag{8}$$

where the superscript c means centre and Δ, δ and u are the corresponding centred variables. On account of (1a) it is seen that Δ is a diagonal matrix. The following notations are now introduced: $\tilde{x}$ - diagonal matrix whose diagonal is x^c, $\bar{\Delta}$ - vector whose elements are Δ_{ii} of Δ, $\tilde{R}_a = (\tilde{R}_{a,1},\ldots,\tilde{R}_{a,n})$ - radius of $\tilde{\Delta}$, i.e. $\tilde{R}_{a,i} = 0.5(\overline{\tilde{\Delta}_i} - \underline{\tilde{\Delta}_i})$, R_b - radius of b, R - the radius of the unknown (approximate) solution U, $C = (A^c)^{-1}$, $\tilde{C} = C\tilde{x}$, R_a - diagonal matrix whose diagonal is formed by the elements of the vector $\tilde{R}_a$. It can be proved that R is the solution of the following real linear system

$$(E - |C|R_a)R = |\tilde{C}|(\tilde{R}_a + R_b) \tag{9}$$

where E is the unit matrix. Finally, the approximate solution X of (8) is the interval vector in centred form

$$X = x^c + [-R, R] \tag{10}$$

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It can be shown that X contains the exact interval solution X^* to (8).

4 A numerical example

To illustrate the applicability of the new method, an electrical circuit contain transistor and diode (Example 6.2 in Reference 4) is considered. Vector-function $f(x)=(f_1(x_1)\ f_2(x_2)\ f_3(x_3))^T$, matrix A and vector b from (2a) are given by

$$f_1(x_1) = 10^{-9}(e^{38x_1} - 1)$$

$$f_2(x_2) = 1.98.10^{-9}(e^{38x_2} - 1)$$

$$f_3(x_3) = 10^{-9}(e^{38x_3} - 1)$$

$$A = \begin{bmatrix} 0.6689 & -1.6722 & 0.6689 \\ 0.6622 & 1.3455 & 0.6622 \\ 1 & -1 & 4 \end{bmatrix}$$

$$b = \begin{bmatrix} 8.0267 \\ -4.0535 \\ 6 \end{bmatrix}$$

The tolerances in the diagonal elements of matrix A are chosen to be 20%. Using the algorithm described in section 3.1 and the simple method for solving the linear interval tolerance problem from 3.2, the following interval vector X^*

$$X^* = \begin{bmatrix} 0.5103 & 0.5778 \\ -4.3520 & -2.6756 \\ 0.3483 & 0.4898 \end{bmatrix}$$

is obtained. The number of iterations for Procedures 1 and 2 are respectively 85 and 81 while the total number of iterations is 166. The interval vector X^* contains (as tightly as possible) the solution set S of (2).

5 Conclusion

We present a new iterative interval method applicable to both dc and ac worst-case tolerance analysis of non-linear circuits. The method is based on description (2) where only part of the diagonal elements of the matrix are intervals. A main feature of the method is the new linearization technique (6) used at each iteration of the computation process. The fact that only part of the diagonal elements of matrix A are interval allows to develop a simple method for approximate solution of the arising linear interval system (7). The numerical efficiency of the method proposed can be improved if sparse matrix techniques are used in solving system (9). This improvement will be substantial for large-scale circuits when the size of the corresponding system (9) is rather high.

6 Acknowledgments

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