Application of global optimization to the design of pipe networks

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ABSTRACT: This paper presents an approach to the optimal design of pipe networks for water distribution. Design is important it often comprises major part of the whole investment in such a system. The problem is solved using a global optimization tool with various random search algorithms and a network simulation model that can handle both static and dynamic loading conditions. An appropriate interface between the two tools performs the decoding of the potential solutions into pipe networks for construction and calculates the corresponding network costs. Two algorithms, adaptive cluster covering and genetic algorithm, yielded promising solutions enabling a choice between accuracy and required computer time. The proposed optimization setup can handle any type of loading condition and neither makes any restriction on the type of hydraulic components in the network nor does it need analytical cost functions for the pipes.

1 INTRODUCTION

A water distribution network is a system containing pipes, reservoirs, pumps, valves of different types, which are connected to each other to provide water to consumers. It is a vital component of the urban infrastructure and requires significant investment.

The problem of optimal design of water distribution networks has various aspects to be considered such as hydraulics, reliability, material availability, water quality, infrastructure and demand patterns. Even though each of these factors has its own part in the planning, design and management of the system and despite their inherent dependence, it is difficult to carry out the overall analysis. Previous research indicates that the formulation of the problem on a component basis is worth doing.

In the present study, the problem is posed as a multi-extremum (global) optimization.

This paper deals with the determination of the optimal diameters of pipes in a network with a predetermined layout. This includes providing the pressure and quantity of water required at every demand node. An appropriate interface is created between a global optimization tool GLOBE (see the accompanying paper of Solomatine 1998) with various random search algorithms, and a network simulation model, EPANET (Rossman 1993), that can handle steady as well as dynamic loading conditions.

2 EXTENT OF THE PROBLEM

The problem reduced to such an extent has two constraints from hydraulic requirements. The continuity constraint states that the discharge into each node must be equal to that leaving the node, except for storage nodes (tanks and reservoirs). This secures the overall mass balance in the network. For *n* nodes in the network, this constraint can be written as

$$\sum_{i=1}^{n} Q_i = 0 \tag{1}$$

where Q_i represents the discharges into or out of the node *i* (sign included).

The second hydraulic constraint is the energy constraint according to which the total head loss around any loop must add up to zero or is equal to the energy delivered by a pump if there is any:

$$\sum h_f = E_p \tag{2}$$

where h_f is the headloss due to friction in a pipe and E_p is the energy supplied by a pump. This embeds the fact that the head loss in any pipe, which is a function of its diameter, length and hydraulic properties, must be equal to the difference in the nodal heads. This constraint makes the problem highly non-linear owing to the nature of the equation that relates frictional head loss and flow. The equation can generally be written as

$$h_f = \frac{aQ^b}{D^c} \quad (3)$$

where a is coefficient depending on length, roughness, etc. b is discharge exponent and c is exponent of pipe diameter (D) which is very close to 5 in most headloss equations.

Considering the diameters of the pipes in the network as decision variables, the problem can be considered as a parameter optimization problem with dimension equal to the number of pipes in the network. Market constraints, however, dictate the use of commercially available (discrete) pipe diameters. With this constraint the problem can be formulated as a combinatorial optimization problem.

The minimum head requirement at the demand nodes is taken as a constraint for the choice of pipe diameters.

Even though the use of an exhaustive search guarantees finding the global optimum, the fact that the computational time increases exponentially with the dimension of the problem makes it impractical to apply them in a multimodal function like this, and especially for real life-size problems.

3 REVIEW OF PREVIOUS RESEARCH

Various researchers have addressed this problem in a number of different ways during the past decades.

Although enumeration techniques (explicit and implicit) are reliable global search methods (Yates et al. 1984, Gessler 1985) their application to practical size networks is limited due to the extraordinarily wide search space and consequently the enormous computational time.

Kessler & Shamir (1989) used the linear programming gradient (LPG) method as an extension of the method proposed by Alperovits & Shamir (1977). It consists of two stages: an LP problem is solved for a given flow distribution and then a search is conducted in the space of flow variables. Later Fujiwara & Khang (1990) used a two-phase decomposition method extending that of Alperovits & Shamir (1977) to non-linear modelling. Also Eiger et al. (1994) used the same formulation as Kessler & Shamir (1989), which leads to the determination of lengths of one or more segments in each link with discrete diameters.

Even though split pipe solutions obtained in the above cases are cheaper, some of the results obtained were not practical and some others were not feasible. In addition to this, some of the methods impose a restriction on the type of the hydraulic components in the network. For instance, the presence of pumps in the network increases the non-linearity of the problem and as a result networks with pumps can not be solved by some of the methods.

Recently genetic algorithms (Goldberg 1989, Michalewicz 1996) have been applied in the problem of pipe network optimization. Simpson & Goldberg (1994), Dandy et al. (1993), Murphy et al. (1994) and Savic & Walters (1997) applied both simple genetic algorithm (SGA) and improved GA, with various enhancements based on the nature of the problem, and reported promising solutions for problems from literature.

Some problems associated with GAs are the uncertainty about the termination of the search and, as in all random search methods, the absence of guarantee for the global optimum.

4 TOOLS USED

4.1 *Optimization tool GLOBE*

GLOBE (Solomatine 1995, 1998, http://www.ihe.nl/hi) is a global optimization tool that incorporates various search algorithms. It iteratively runs an executable program that receives potential solutions generated by the search algorithms and returns a corresponding value of the objective function. Out of the various optimization algorithms implemented in GLOBE, the following four are used on the problem: Controlled Random Search (CRS2) (Price 1983), CRS4 (Ali & Storey, 1994), Genetic Algorithm (Goldberg 1989) and Adaptive Cluster Covering with Local Search (ACCOL) (Solomatine 1998).

4.2 *Network simulation tool - EPANET*

The network simulation model used is EPANET (Rossman 1994, www.epa.gov). It calculates nodal heads and flows in pipelines, storage in each tank, concentration of substance throughout the system, and water age and source tracing both at static and dynamic loading conditions.

Hazen-Williams equation is used due to its wide applicability in water supply networks. This equation can be written as

$$h_f = 4.72 C^{-1.85} D^{-4.87} L \quad (4)$$

where h_f is head loss in the pipe, C is Hazen-Williams roughness coefficient, D is pipe diameter in ft and L is length of the pipe in ft.

5 PROBLEM FORMULATION

5.1 *Constraint handling*

The constraints in the problem can be grouped into the following: hydrodynamic, minimum head and commercial.

The hydrodynamic constraints are handled by the network simulation model.

The optimization package handles only box type constraints on the parameters i.e. upper and lower bounds on each parameter. Penalty functions are used to handle minimum nodal head constraints.

Commercial constraints reduce the parameter space to a discrete one. GLOBE has an option to fix the resolution of the parameter space to be searched. This can be adjusted to the number of available commercially available pipe sizes and each parameter can take values from one to the number of commercial pipe sizes. This number is used as an index for the choice of diameters, therefore, the search algorithms will search for the optimal set of pipe indices instead of the optimal set of diameters. This approach has the following technical advantages:

1. the search algorithms will not spend computer time looking for diameters in a real parameter space, and
2. the solutions obtained will not be split pipe solutions.

5.2 *Objective function*

The objective function to be minimized by the optimization algorithms is the cost of the network. If the actual cost of the network is the sole objective function, then obviously the search will end up with the minimum possible diameters allocated to each of the pipes in the network. To tackle this, a penalty cost is added to the actual cost of the network based on the minimum head constraint.

5.2.1 *Actual cost of the network*

The actual cost of the network C_a is calculated based on the cost per unit length associated with the diameter and the length of the pipe:

$$C_a = \sum_{i=1}^{n} c(D_i) L_i \tag{5}$$

where n is the number of pipes in the network and $c(D_i)$ is the cost per unit length of the i^{th} pipe with diameter D_i and length L_i.

5.2.2 *Penalty cost*

The penalty cost is superimposed on top of the actual cost of the network in such a way that it will discourage the search in the infeasible direction. It is defined on the basis of the difference between the required minimum head *(H_{min})* at the demand nodes and the lowest nodal head obtained after simulation. It depends upon the degree of pressure violation and the cost of the network in some cases and is defined in the following way:

1. For networks in which all the nodal heads are greater than H_{min} the penalty cost is zero.
2. For networks in which the minimum head is greater than zero but less than H_{min} it increases linearly with the nodal head deficit. i.e,

$$C_p = p \times C_{\max} \times \underset{i=1ton}{Max}\left(H_{\min} - H_i\right) \tag{6}$$

where p is a penalty coefficient and C_{max} is the maximum possible cost that the network can have (calculated on the cost of the largest commercial pipe available).

3. When the network is composed of pipes with very small diameters, the nodal heads obtained from the simulation will be very large negative numbers, which in some cases cause computational overflow problems. Therefore, for networks in which the minimum head falls below zero, the penalty cost is defined as a very high cost minus twice the cost of the network. This is done to provide a slope towards the choice of larger pipe diameters so that the search algorithms can get some heuristic clue about the objective function. This penalty should obviously be greater than that of case (2).

$$C_p = 2 \times p \times C_{\max} - 2 \times C_a \tag{7}$$

5.3 *Working Algorithm of the Cost Function*

The following steps are used to calculate the cost of one network (Fig. 1):

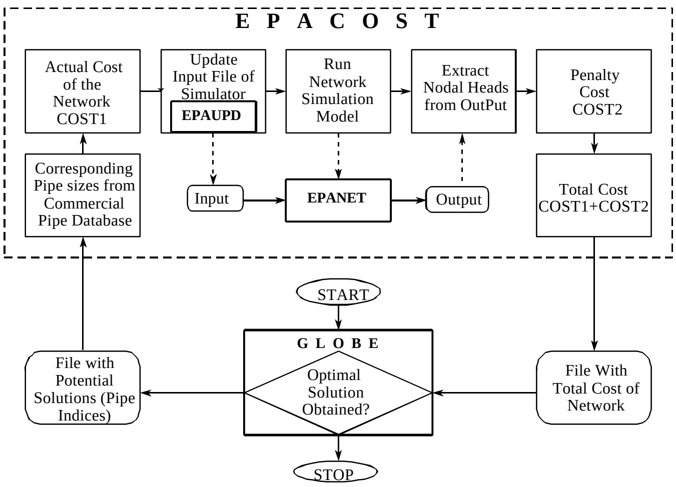

Figure 1. Problem setup.

1. Numbers generated by GLOBE are read from the parameter file and converted to indices of pipe sizes that represent one network.
2. The actual cost of the network (Cost 1) is calculated based on the length and cost per unit length corresponding to the diameter of each pipe.
3. The input file of the simulator is updated (only the diameters are changed).
4. The network simulation model is run.

5. From the output file of the simulation, the nodal heads are extracted and the minimum head is identified.
6. Penalty cost (Cost 2) is calculated based on the degree of nodal head violation if any.
7. The total cost of the network (Cost 1 + Cost 2) is passed to the response file.

6 THE TEST PROBLEM

The test problem is a two-loop network with 8 pipes, 7 nodes and one reservoir (Fig. 2) which is obtained from the literature (Alperovits & Shamir, 1977). All the pipes are 1000 m long and Hazen-Williams coefficient is assumed to be 130 for all the pipes. The minimum nodal head requirement for all demand nodes is 30m. There are 14 commercially available pipe diameters (Table 1).

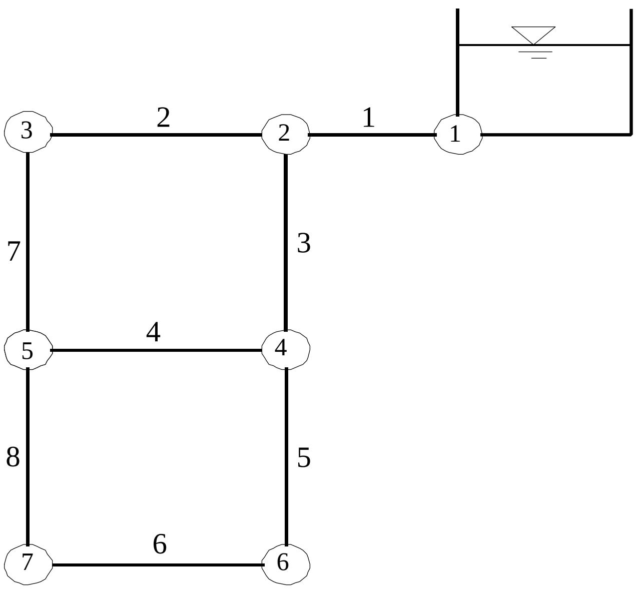

Figure 2. The two-loop network

Table 1. Cost data for the two-loop network

Diameter (inches)	Cost (units)
1	2
2	5
3	8
4	11
6	16
8	23
10	32
12	50
14	60
16	90
18	130
20	170
22	300
24	550

Table 2. Node data for the two-loop network

Node	Demand (m^3/hr)	Ground level (m)
1 (Reservoir)	-1,120.0	210.00
2	100.0	150.00
3	100.0	160.00
4	120.0	155.00
5	270.0	150.00
6	330.0	165.00
7	200.0	160.00

Table 3. Optimal pipe diameters (inches) for the two-loop network

Pipe No.	Algorithm				
	CRS2	GA	ACCOL	CRS4	Best run
1	18	18	22	18	18
2	10	14	18	16	10
3	16	14	20	14	16
4	4	1	3	2	4
5	16	14	16	14	16
6	10	1	4	1	10
7	10	14	18	14	10
8	2	12	16	10	1
Cost (units)	422000	424000	447000	439000	419000
Evaluations	10009	3381	1810	720	1373
Fraction of total space	6.78e-6	2.29e-6	1.23e-6	4.9e-7	9.3e-7

Table 4. Nodal heads (m) corresponding optimal diameters

Node	Algorithm				
	CRS2	GA	ACCOL	CRS4	Best run
1	0.00	0.00	0.00	0.00	0.00
2	53.21	53.21	57.45	53.21	53.21
3	30.50	36.62	45.59	39.79	30.34
4	43.36	43.92	51.65	43.89	43.39
5	33.92	42.01	54.31	45.22	33.63
6	30.30	31.51	40.32	31.47	30.36
7	30.25	30.01	42.86	30.34	30.43

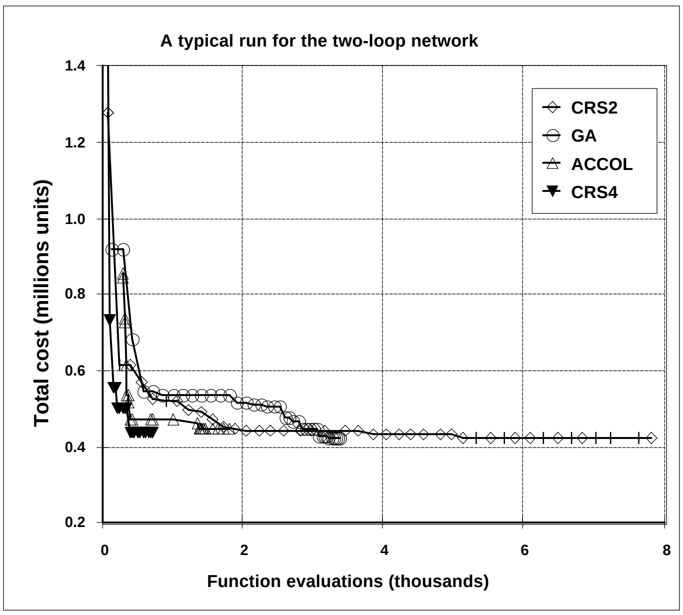

Figure 3.

Table 5. Results obtained for the two-loop network in previous research

Pipe	Alperovits & Shamir (1977)		Goulter et al. (1986)		Kessler & Shamir (1989)		Eiger et al. (1994)		Savic & Walters* (1997)	
									GA 1	GA 2
	L (m)	D(in)	L (m)	D (in)	L (m)	D (in)	L (m)	D (in)	D (in)	D (in)
1	256.00	20	383.00	20	1000.00	18	1000.00	18	18	20
	744.00	18	617.33	18						
2	996.38	8	1000.00	10	66.00	12	238.02	12	10	10
	3.63	6			934.00	10	761.98	10		
3	1000.00	18	1000.00	16	1000.00	16	1000.00	16	16	16
4	319.38	8	687.00	6	713.00	3	1000.00	1	4	1
	680.62	6	313.00	4	287.00	2				
5	1000.00	16	1000.00	16	836.00	16	628.86	16	16	14
					164.00	14	371.14	14		
6	784.94	12	98.00	12	109.00	12	989.05	10	10	10
	215.06	10	902.00	10	891.00	10	10.95	8		
7	1000.00	6	492.00	10	819.00	10	921.86	10	10	10
			508.00	8	181.00	8	78.14	8		
8	990.93	6	20.00	2	920.00	3	1000.00	1	1	1
	9.07	4	980.00	1	80.00	2				
Cost (units)		497525		435015		417500		402352	419000	420000

* These two columns contain the results reported by Savic & Walters (1997) using different numerical conversion constants for the head loss equation.

According to the results obtained from the two-loop network, CRS4 and ACCOL are found to be fast converging and CRS2 is generally much slower. GA is slower than the first two but is generally much faster than CRS2. Regarding accuracy (minimization of cost), the minimum cost obtained in every run does not belong to the same algorithm, however, near optimal values are obtained with all the algorithms for the two-loop network.

Optimal pipe diameters, the resulting cost of network and the number of function evaluations for each algorithm in one typical run and the best run are shown in Table 3. Table 4 shows the corresponding nodal heads obtained as a result of simulation. In all cases the minimum nodal head requirement is not violated.

It can be observed from Table 3 that only a small fraction of the total search space is searched by each algorithm. It is known that for the two-loop network containing 8 pipes and with 14 available commercial pipe sizes the total number of possible combinations is 14^8 which is nearly 1.5 billion.

It is observed that all the algorithms stopped near the optimum, in all cases with feasible and single diameter solutions. Moreover, the solutions obtained by the different algorithms represent entirely different pipe networks with a small variation in cost. This indeed provides alternatives for decision-makers and implicitly resolves some objectives that cannot be enumerated during the optimization process.

As can be observed from Table 3, the best single diameter solutions for the two loop network has been reproduced within nearly 7 minutes on a computer with Pentium 100MHz processor.

7 APPLICATION TO NEW PIPE NETWORK DESIGN

The problem that is considered for a new pipe network design is the Hanoi Network (water supply network of Hanoi, Vietnam). Data obtained from literature (Fujiwara & Khang, 1990) are used. The network (Fig. 5) contains 34 pipes, 31 demand nodes and a reservoir.

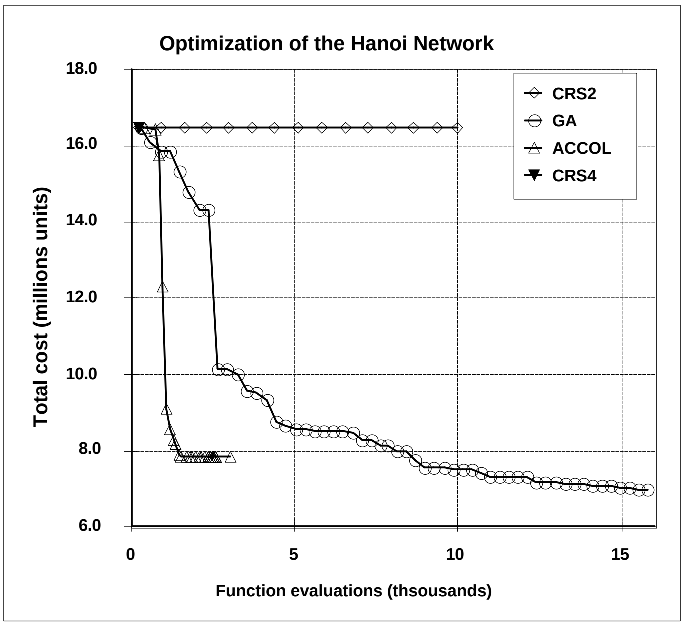

Figure 5.

Table 6. Diameters and cost data for the Hanoi network.

Diameter (inches)	Cost per unit length (units)
12	45.73
16	70.40
20	98.39
24	129.33
30	180.75
40	278.28

The minimum nodal head required at all demand nodes is 30m. Diameters of commercially available pipes used and their costs per unit length are shown in Table 6. The cost per unit length is calculated based on the analytical cost function $1.1 \times D^{1.5}$ used by Fujiwara & Khang (1990). It must be noted that our approach does not require having an analytical function relating the cost per unit length of the pipes to the diameter.

Table 7. Optimal diameters (Hanoi network)

Pipe Number	Diameter (inches)	
	GA	ACCOL
1	40	40
2	40	40
3	40	40
4	40	40
5	30	40
6	40	30
7	40	40
8	30	40
9	30	24
10	30	40
11	30	30
12	30	40
13	16	16
14	24	16
15	30	30
16	30	12
17	30	20
18	40	24
19	40	30
20	40	40
21	20	30
22	20	30
23	30	40
24	16	40
25	20	40
26	12	24
27	24	30
28	20	12
29	24	16
30	30	40
31	30	16
32	30	20
33	30	30
34	12	24
Cost (millions)	7.0	7.8
Evaluations	16910	3055

The optimization was carried out using the algorithms mentioned above and the results obtained from one representative run are indicated graphically (Fig. 6) and in tabular form (Tables 7 and 8).

It is observed that the algorithm CRS2 terminated without any improvement in the network obtained by the first iteration. It stopped after exhausting the total number of function evaluations allowed. On the other hand, CRS4 in almost all of the experiments stopped after exhausting the initial iteration. One possible reason for the failure of these algorithms is that they were designed for continuous variables.

Table 8. Nodal heads (Hanoi network)

Node number	Nodal heads (m)	
	GA	ACCOL
1 (Reservoir)	00.00	00.00
2	97.14	97.14
3	61.67	61.67
4	58.59	57.68
5	54.82	52.75
6	39.45	47.65
7	38.65	42.97
8	37.87	41.68
9	35.65	40.70
10	34.28	32.46
11	32.72	32.08
12	31.56	30.92
13	30.13	30.56
14	36.36	30.55
15	37.17	30.69
16	37.63	30.74
17	48.11	46.16
18	58.62	54.41
19	60.64	60.58
20	53.87	49.23
21	44.48	47.92
22	44.05	47.86
23	39.83	41.96
24	30.51	40.18
25	30.50	38.95
26	32.14	36.01
27	32.62	35.93
28	33.52	36.47
29	31.46	36.45
30	30.44	36.54
31	30.39	36.64
32	30.17	36.76

Genetic algorithm and ACCOL moved the search towards the global munimum. For GA it took relatively more function evaluations (about 17000, 1hr and 15 minutes on Pentium 100MHz PC) and ended up with a better least cost solution. ACCOL on the other hand converged several times faster (about 3000 function evaluations, 17000, which is 15 minutes on the same PC) and reported a solution slightly more expensive (11%) than that of Genetic Algorithm. The results obtained from the two algorithms are feasible from the nodal head point of view.

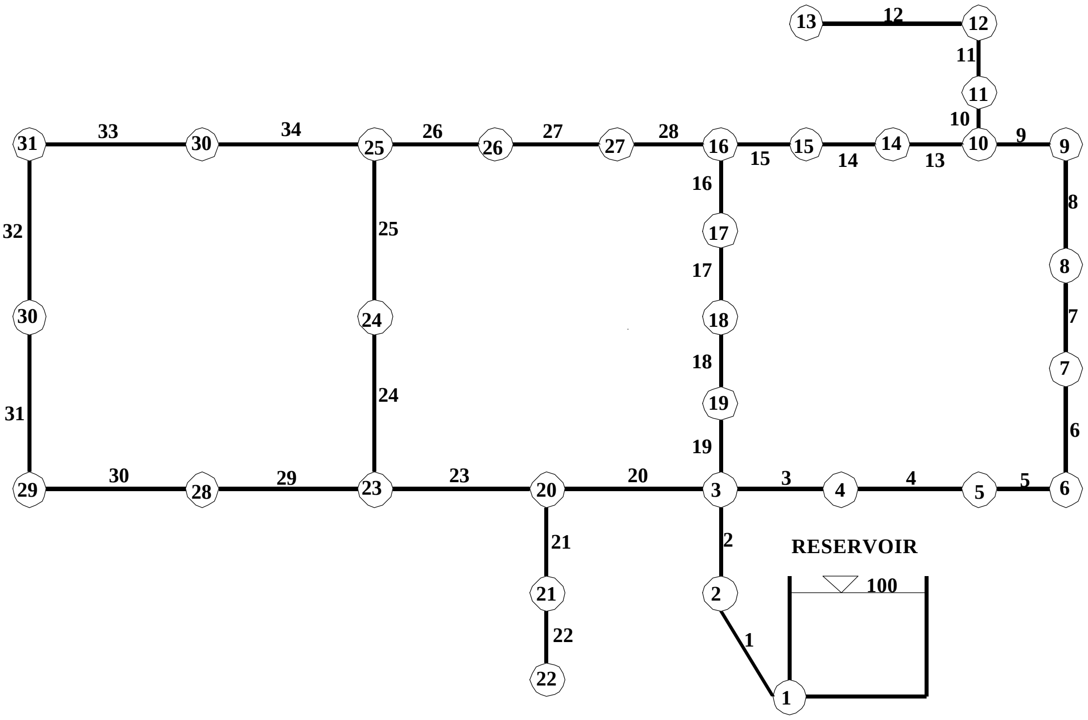

Figure 5. The Hanoi network.

Table 9. Comparison of the solution obtained for the Hanoi network with previous research.

	Fujiwara & Khang (1990)	Fujiwara & Khang (1990)	Eiger et al. (1994)	Savic & Walters (1997)		This paper	
				GA No. 1	GA No. 2	GA	ACCOL
Continuous solution*	Yes	No	No	Yes	Yes	Yes	Yes
Feasible**	No	No	No	Yes	Yes	Yes	Yes
No. of nodes with head deficit	18	18	6				
Cost of network (millions)	5.354	5.562	6.027	6.073	6.195	7.006	7.836

* "No" implies split pipe solution.

** Only in terms of nodal head violation.

Optimal diameters of the pipes and their corresponding nodal heads obtained by simulation are tabulated in Tables 7 and 8.

8 DISCUSSION AND CONCLUSION

GA and ACCOL algorithms showed their efficiency and effectiveness. On Hanoi network GA found the solution with lower (10%) cost than ACCOL, but required 3 to 5 times more function evaluations (model runs). For large networks it is sensible to use a suite of algorithms in order to have the choice between fast-running algorithms and algorithms oriented towards more exhaustive but longer search.

The fact that the problems considered do not have pumping facilities is simply because these problems were taken from literature. It is however possible to optimize networks with any kind of hydraulic facilities as long as the network simulator is capable of handling it.

Since global optimisation methods work with any objective (cost) functions, they can also be efficiently used to optimize not only design but also operation, maintenance and other aspects of water distribution.

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