



# A new parallel method for verified global optimization

Zur Erlangung des akademischen Grades eines

Doktors der Naturwissenschaften

am Fachbereich Mathematik der  
Bergischen Universität Gesamthochschule Wuppertal  
genehmigte

Dissertation

von

Suiunbek Ibraev

aus Bischkek (Kirgisien)

Tag der mündlichen Prüfung: 17. Juli 2001  
Referent: Prof. Dr. A. Frommer  
Korreferent: Prof. Dr. W. Krämer



---

# Contents

---

<b>Introduction</b>	<b>vii</b>
<b>1 Background</b>	<b>1</b>
1.1 Terms and Algorithm Notations . . . . .	1
1.2 The Problem of Global Optimization . . . . .	3
1.2.1 Classical Methods . . . . .	3
1.2.2 Interval Methods . . . . .	4
1.2.3 Interval Arithmetic . . . . .	6
1.2.4 Machine Arithmetic . . . . .	9
1.2.5 Inclusion Functions . . . . .	10
1.2.6 The Natural Interval Evaluation . . . . .	10
1.2.7 The Mean Value and Taylor Forms . . . . .	11
1.2.8 The Order of the Inclusion Function . . . . .	12
1.3 Differentiation . . . . .	12
1.4 The Programming Language in Use . . . . .	13
<b>2 The Serial Method for Verified Global Optimization</b>	<b>15</b>
2.1 Strategies for the Box Selection . . . . .	17
2.1.1 <i>oldest-first</i> -Strategy . . . . .	17
2.1.2 <i>best-first</i> -Strategy . . . . .	17
2.1.3 <i>depth-first</i> -Strategy . . . . .	18
2.2 Comparison of Box Selection Strategies . . . . .	18
2.3 Strategies for the Box Subdivision . . . . .	19
2.3.1 Bisection Strategies . . . . .	19
2.3.2 Multisection Strategies . . . . .	21
2.4 Accelerating Devices . . . . .	22

2.4.1	The Monotonicity Test . . . . .	23
2.4.2	The Nonconvexity Test and the Interval Krawczyk Method . . . . .	23
2.5	A New Serial Method for Verified Global Optimization . . . . .	25
2.5.1	Modification of the Krawczyk Method . . . . .	25
2.5.2	A New Strategy for Box Processing . . . . .	26
2.5.3	A New Strategy for Applying the Krawczyk Method . . . . .	27
2.5.4	Dependence of the results on the differentiation technique . . . . .	27
2.5.5	A New Serial Method for Verified Global Optimization . . . . .	28
<b>3</b>	<b>Nonlinear Systems: Convergence and Divergence of Interval Newton and Krawczyk Methods</b>	<b>41</b>
3.1	Notations and Preliminaries . . . . .	41
3.2	Quadratic Divergence of the Newton Method . . . . .	53
3.3	The Simplified Newton Method . . . . .	54
3.4	Linear divergence of the Simplified Newton Method . . . . .	54
3.5	The Krawczyk Method . . . . .	57
3.6	The Simplified Krawczyk Method . . . . .	58
3.7	The Modified Krawczyk Method (with LU Factorization) . . . . .	65
3.8	The Simplified Modified Krawczyk Method (with LU Factorization) . . . . .	66
3.9	Numerical examples . . . . .	73
<b>4</b>	<b>The Parallel Method for Verified Global Optimization</b>	<b>79</b>
4.1	Parallel Processing . . . . .	79
4.1.1	Architecture Classifications . . . . .	80
4.1.2	Measures of Performance . . . . .	83
4.2	Existing Parallel Approaches . . . . .	83
4.2.1	The Approach of Dixon and Jha (1993) . . . . .	85
4.2.2	The Approach of Henriksen and Madsen (1992) . . . . .	85
4.2.3	The Approach of Eriksson (1991) . . . . .	87
4.2.4	The Approach of Moore, Hansen and Leclerc (1992) . . . . .	88
4.2.5	The Approach of Berner (1995) . . . . .	89
4.2.6	The Approach of Wiethoff (1997) . . . . .	91
4.3	A New Parallel Approach: a Challenge Leadership Model . . . . .	91
4.3.1	General Conditions for the Parallelization . . . . .	91
4.3.2	Description of the Communication Model . . . . .	92
4.3.3	Used Communication Routines . . . . .	93
4.3.4	A New Parallel Method . . . . .	94
4.3.5	Numerical Results . . . . .	102
4.3.6	Comparison . . . . .	107

4.3.7 A Problem from Industry . . . . .	110
<b>A Considered Test Problems</b>	<b>113</b>
A.1 Simple Problems . . . . .	113
A.2 Medium Problems . . . . .	118
A.3 Hard Problems . . . . .	123
A.4 A Problem from Industry . . . . .	128
<b>Literature</b>	<b>133</b>



---

# List of Tables

---

2.1	The use of Algorithm 2.7 instead of Algorithm 2.6 gives a little improvement . . . . .	29
2.2	Different strategies for processing a box by subdivision after the interval Krawczyk method . . . . .	31
2.3	Time measured with different $t$ . . . . .	32
2.4	Time measured with different $t$ for medium and hard problems only . . . . .	33
2.5	Applying the adaptive approach to the algorithm with the interval Newton method . . . . .	34
2.6	Comparison of Krawczyk and Newton methods using the adaptive approach with the algorithm by Wiethoff (with weighted percentage) . . . . .	35
2.7	The use of Algorithm 2.7 instead of Algorithm 2.6 gives a little improvement (using automatic differentiation) . . . . .	35
2.8	Different strategies for processing the box by subdivision after the interval Krawczyk method (using automatic differentiation) . . . . .	36
2.9	Time measured with different $t$ (using automatic differentiation) . . . . .	37
2.10	Applying the adaptive approach to the algorithm with the interval Newton method (using automatic differentiation) . . . . .	38
2.11	Comparison of Krawczyk and Newton methods using the adaptive approach with Wiethoff's method (with weighted percentage, using automatic differentiation) . . . . .	39
3.1	Time measurements for Example 1 . . . . .	74
3.2	Time measurements for Example 2 with $\phi(u) = \alpha \cdot e^u$ , $[u] = [-1, 0]$ . . . . .	75
3.3	Time measurements for Example 2 with $\phi(u) = u^4 + u^3 + 1$ , $\alpha = 1$ . . . . .	77
4.1	Times for test problems on a cluster of Sun workstations using the challenge leadership approach . . . . .	103
4.2	Number of function, gradient, Hessian evaluations by the serial and parallel methods . . . . .	104
4.3	Times for test problems on a cluster of Sun workstations using centralized mediator approach . . . . .	107