Global Optimization in Several Operations Research Problems

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In our report, we consider the connections between theory and methods of global optimization with regard to the following problems.

1. To find global lower and upper bounds for optimal values of linear programs with interval coefficients in data.

2. The inverse problem for interval linear programming, i.e., the problem of finding the coefficients from the prescribed intervals, such that a given vector is an optimal solution of the corresponding linear program.

3. Certain special semi-infinite programming problems.

4. Finding a Nash equilibrium in n-person games with loss functions of special types.

5. Compromise functioning of power energy systems.

Different global optimization methods are described and numerical results of their application are given.

Optimization of Estimates for Characteristics of Dynamical Systems Based on 2nd Lyapunov Method

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Many problems of investigating stability and calculating characteristics of dynamical systems (time and value of transient processes, speed of convergence, stability region, integral criteria, etc.) are solved using direct Lyapunov method. The method is based on searching the function having certain properties along the system solutions. It is essential that if there exists at least one Lyapunov function then (due to continuity) there may be a lot of such functions. If we formulate the problem to construct the function that estimates the process characterictics in the best way then we get an optimization problem in the functional space.

The best results can be obtained using Lyapunov functions of the fixed form. In such a case the problem is reduced to parametric optimization in a finite-dimensional space. For Lyapunov function they often use quadratic forms with positive definite matrices $v(x) = x^T Hx$, $x \in R^n$. In particular, for linear systems of ordinary differential equations

$$\dot{x}(t) = Ax(t), \quad x(t_0) = x_0,$$

with the aid of H one can get the functions

$$\phi_1(H) = \frac{\lambda_{\min}(H)}{\lambda_{\max}(H)}, \quad \phi_2(H) = \frac{\lambda_{\min}(-A^T H - HA)}{\lambda_{\max}(H)} \sqrt{\frac{\lambda_{\min}(H)}{\lambda_{\max}(H)}},$$

$$\phi_3(H) = \frac{\lambda_{\min}(-A^T H - HA)}{\lambda_{\max}(H)} \ln \left[\sqrt{\frac{\lambda_{\min}(H)}{\lambda_{\max}(H)}} \cdot \frac{|x_0|}{\varepsilon} \right],$$

characterising the estimate of the regulation value, the integral criterion, and the time of transient processes. Here $\lambda_{\min}(\cdot)$, $\lambda_{\max}(\cdot)$ are extremal eigenvalues of corresponding matrices.

The problem of search for optimal estimates can be reduced to optimization problems with nonsmooth target functions on the set $L(H)$ being cone of positive definite matrices. In series of cases the target functions can be replaced by convex ones and they can result in convex programming. There are obtained algorithms of implementation based on generalized gradient method and derivatives along a direction.

In sequal there are considered problems of search for stability conditions and optimal estimation of characteristics of differential systems with delay

$$\dot{x}(t) = Ax(t) + Bx(t - \tau), \quad \tau > 0,$$

and difference systems with delay

$$x(k+1) = Ax(k) + Bx(k - m), \quad m > 0.$$

Optimal Strategies in the Problem of Allocating State Production Orders

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In this paper, the problem of control of diversified industrial complex with the use of allocating state production orders is considered. The interaction between a control institution and enterprises is described by two-level hierarchical system. The lower level of the system represents the enterprises of the industrial complex. The higher level represents the control institution.

We assume that the control institution has a budget B, which needs to be allocated among enterprises in the form of orders for production subject to constraints

$$0 \le C_i^g \le C_i^m, \tag{1}$$

$$\sum_{i=1}^{n} p_i C_i^m > \frac{B}{T}, \tag{2}$$

where C_i^g and C_i^m are the ordered amount and the maximal demand for the ith enterprise production, respectively, and T is the planning horizon.

We suppose that the importance of the ith type of production is determined by the weight λ_i^g and the weight of importance of the jth enterprise development for the economy is λ_i^p. In this case, the goals of control institution are to maximize the overall utility of ordered production, on the one hand, and to promote development of the most important enterprises, on the other hand. As an effectiveness criterion we propose the convolution of these goals

$$J_y(C^g) = \sum_{i=1}^{n} \lambda_i^g C_i^g + \sum_{i=1}^{n} \lambda_i^p K_i(T) \to \max, \tag{3}$$

where $K_i(T)$ is the amount of capital of the i-th enterprise at the moment T, which is determined by dynamic model of industrial complex with the consumers' demand

$$C = C^g + C^e,$$

where C^e is a demand of external economic agents. Thus, the problem of allocating government production orders can be formulated in the following form: to find the values of $C^g = (C_1^g, C_2^g, \ldots, C_n^g)$ satisfying the inequalities (1), (2), which maximize criterion (3). The optimal allocation for this problem is established by the following theorem.

Theorem. Let's consider vector $\lambda = (P^{-1})'(\lambda^g + ((E-H)^{-1})'\lambda^p)$, where P is the diagonal matrix of enterprise production prices p_i. Let $\lambda^* = (\lambda_{i_1}, \lambda_{i_2}, \ldots, \lambda_{i_n})$ be the vector λ with lexicographically ordered elements. Let k be the index $1 \le k \le n$ such that

$$\sum_{l=k+1}^{n} p_{i_l} C_{i_l}^m \le \frac{B}{T}; \qquad \sum_{l=k}^{n} p_{i_l} C_{i_l}^m > \frac{B}{T}.$$

Then the function $J_y(C^g)$ achieves its maximum on the set defined by (1), (2) at the vector with the components

$$C_{i_l}^{g*} = \begin{cases} C_{i_l}^m, & l > k, \\ \frac{1}{p_{i_k}}(\frac{B}{T} - \sum_{j=k+1}^n p_{i_j} C_{i_j}^m), & l = k, \\ 0, & l < k, \end{cases} \qquad l = 1, \ldots, n. \tag{4}$$

Thus, the optimal solution of (1)-(3) could be found by the following algorithm:

1. the set of enterprises is ordered according to the vector λ;
2. the elements of demand vector C^g are calculated using (4).

The number of operations in this algorithm is defined by the procedure of sorting for the vector λ and equal to $O(n \ln(n))$ while using effective methods of sorting. This is much less than the number of operations in standard algorithms of linear programming.

The method allows to find effective solutions of the problem taking into account both relative utilities of different types of production and enterprise importance. It is shown that this solution leads to more effective resources allocation than non-cooperative interaction does.

Scenario Calculus as Effective Tool for Scenario Analysis

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We are going to present a methodology of a new scientific project, which is developing in the Institute of Control Sciences of RAS. One of the most important ideas of the project is to create a new effective semiautomatic tool. This tool is expected to be used for analysis and synthesis of complex social and economical systems. It allows to construct advising expert systems, which automate a decision-making process. The new principle called scenario methodology is developed in order to create this tool.

Decision-making assumes that the information describes the analyzed system from different sides, including data of its creation, functionality, and developing. There are two special problems of the decision-making process. The first problem is to represent the information as a sequence of events. Note that decision-makers analyze and choose the effective control with the help of their own professional language. Therefore, the other problem is to interpret their idea and to analyze the situation in a proper way. We suggest a methodology to solve the problems. It is based on the creating a spectrum of scenarios that define main schemes and directions of possible system development under given hypotheses.

We use the subject-object methodology to describe a behavior of the system object. There are distinguished six modules which are responsible for describing the object: identical object model, environment model, living model, measurement model, measurement environment model, selection orders (selection model), and fixing of the object state (discrete order of living object trajectory). These elements are grouped to general metaset of the system. It allowes to define a notion of scenario of object behaviour as sequences of events which are most important from the point of view of decision-maker.

In this way we formalize an event, situation, scenario, and it's fragments. We represent principles and methods to define topological structures, create scenario characters, and indicate its properties. Two general concepts of control and mathematics directions as opposite points of view have been researched: chaos theory and cybernetics.

The next step is to construct a spectrum of scenario spaces to represent different management fields: technical, technological, organization, economical, lawful, etc. These are scenario operations in scenario space. Thus, scenario calculus is created.

The applied areas are quite wide: a management to provide ecology security; researching sociological and economical systems; scheduling and planning; working out effective technologies for Logistics; creating a strategies to eliminate extreme situations; expending nuclear weapons over the world and other catastrophe in large-scale systems; real steps of financial crises; effective strategies for investments; and others.

Advanced imitation system of decision support based on the scenario methodology has been developed. We intend to discuss analytic and experimental results of our research. Applications are as follows: directive or economical limitation of production as main resource to eliminate ecology pollution, 500 days-economical conception by G. Yavlinsky, analysis of macro-economical parameters, expending the computer technology in Russia, and others.

Management Controls in Telephone Networks

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The traffic in the telephone network is distributed with the help of route tables. The route tables developed on the stage of network synthesis are realized in exchange apparatus and cannot be slightly changed in the process of running. At the same time the network operative situation demands the possibility of sufficiently quick and effective flow control.

Suddenly load change, refusal or damage of network equipment force new connection routs, load limiting in some directions, etc. In fact, it is required to change operative routing without changing route table directly. Such possibility is supplied by additional management controls. The use of these controls allows the following: call blocking and gapping for some code directions; circuit directionalization and selective circuit reservation for some circuit groups; cancel alternating and temporary alternative routing for some flow types.

The strategy of real time management control will be discussed. The decision on the additional management necessity can be taken with a time interval of 15 minutes. The decision scheme includes:

1) identification of the network state proceeding from real observations of the network data;

2) parameterization of the management control;

3) simulation of traffic flows and call connections;

4) optimization of management control parameters;

5) management controls' recommendation.

Algorithms of the parameter optimization are based on the methods of adaptive control of Markovian chains. The results of computing simulation will be also discussed.

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The Equivalence of Different Formulations for the Problem of Constructing Economic Indices

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In this paper, the problem of constructing smooth positive-homogeneous economic indices (utility function and price index) is considered. The equivalence of different formulations of the problem is established through the application of the rationalization conditions for the demand or inverse demand functions in the corresponding cases and the corollary of the Crouzeix duality theorem [1].

At first, we consider the construction of economic indices by virtue of the inverse demand functions.

Definition. *A class $\mathcal{U}_n$ is a set of functions $F(\vec{X})$ that are continuous on R^n_+ and satisfy the following conditions on int R^n_+:*

(O_1) $F(\vec{X}) > 0$ *for any* $\vec{X} > 0$;

(O_2) $F(\vec{X}) \in C_1(\textit{int}\, R^n_+)$;

(O_3) $F(\lambda\vec{X}) = \lambda F(\vec{X})$ *for all* $\lambda > 0$ *and any* $\vec{X} > 0$;

(O_4) $F'(\vec{X}) > 0$ *for all* $\vec{X} > 0$;

(O_5) $F(\vec{X})$ *is strictly quasi-concave;*

(O_6) *for all* $\vec{Y} > 0$, *the minimization problem* $\inf\limits_{\vec{X}>0} \frac{\langle \vec{X}, \vec{Y} \rangle}{F(\vec{X})}$ *has at least one optimal solution in int* R^n_+.

Denote by N the set $\{1, \ldots, n\}$.

Theorem. Suppose the inverse demand functions $\vec{P}(\vec{X})$ are continuously differentiable on R^n_+. The conditions:

(C_1) $\vec{P}(\vec{X}) > 0$ for any $\vec{X} > 0$;

(C_2) for all $i, j \in N$, any $\lambda > 0$ and all $\vec{X} > 0$ the following relation is fulfilled:

$$\frac{P_i(\lambda \vec{X})}{P_j(\lambda \vec{X})} = \frac{P_i(\vec{X})}{P_j(\vec{X})};$$

(C_3) for all $\vec{X}_1$, $\vec{X}_2 > 0$ such that $\vec{X}_1 \neq \lambda \vec{X}_2$ for any $\lambda > 0$

$$\langle \vec{P}(\vec{X}_1), \vec{X}_2 \rangle \langle \vec{P}(\vec{X}_2), \vec{X}_1 \rangle > \langle \vec{P}(\vec{X}_1), \vec{X}_1 \rangle \langle \vec{P}(\vec{X}_2), \vec{X}_2 \rangle;$$

(C_4) for all distinct numbers i, j, $k \in N$ and any $\vec{X} > 0$

$$P_i(\vec{X}) \left(\frac{\partial P_j}{\partial X_k}(\vec{X}) - \frac{\partial P_k}{\partial X_j}(\vec{X}) \right) + P_j(\vec{X}) \left(\frac{\partial P_k}{\partial X_i}(\vec{X}) - \frac{\partial P_i}{\partial X_k}(\vec{X}) \right) +$$

$$+ P_k(\vec{X}) \left(\frac{\partial P_i}{\partial X_j}(\vec{X}) - \frac{\partial P_j}{\partial X_i}(\vec{X}) \right) = 0;$$

(C_5) for all $\vec{X} \in \partial R^n_+$

$$(N \setminus \{i \in N |\, X_i = 0\}) \bigcap \{j \in N |\, P_j(\vec{X}) = 0\} \neq \emptyset$$

are necessary and sufficient for the fulfillment of each of the following conditions:

(F_1) there exists the utility function $F(\vec{Y}) \in \mathcal{U}_n$ such that for all $\vec{X} \geq 0$ it follows that

$$\vec{X} \in \text{Argmax}\{F(\vec{Y}) | \langle \vec{P}(\vec{X}), \vec{Y} \rangle \leq \langle \vec{P}(\vec{X}), \vec{X} \rangle,\ \vec{Y} \geq 0\};$$

(F_2) there exists the price index $q(\vec{s}) \in \mathcal{U}_n$ such that for all $\vec{X} \geq 0$ it follows that

$$\vec{P}(\vec{X}) \in \text{Argmax}\{q(\vec{s}) | \langle \vec{s}, \vec{X} \rangle \leq \langle \vec{P}(\vec{X}), \vec{X} \rangle,\ \vec{s} \geq 0\};$$

(F_3) there exist the utility function $F(\vec{Y}) \in \mathcal{U}_n$ and the price index

$q(\vec{s}) \in \mathcal{U}_n$ such that for all $\vec{X}$, $\vec{p} \geq 0$

$$\begin{cases} q(\vec{p})F(\vec{X}) \leq \langle \vec{p}, \vec{X} \rangle; \\ q(\vec{P}(\vec{X}))F(\vec{X}) = \langle \vec{P}(\vec{X}), \vec{X} \rangle; \end{cases}$$

(F_4) there exist the utility function $F(\vec{Y}) \in \mathcal{U}_n$ and the price index $q(\vec{s}) \in \mathcal{U}_n$ such that for all $\vec{X} \geq 0$

$$q(\vec{P}(\vec{X}))dF(\vec{X}) = \sum_{i=1}^{n} P_i(\vec{X})dX_i.$$

Because of duality it is possible to reformulate the equivalence theorem in terms of the demand functions analogously.

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Reference

1. Kornyushina, D.S., Shananin, A.A. About rationalization of consumer demand in the class of smooth positive-homogeneous utility functions // Proceedings of school-symposium "Mathematical modelling in natural sciences and the humanities" (Voronezh, January 20-27, 2000). Voronezh: Voronezh State University, 2000. P.74-82 (in Russian).

Methods of Schedule Construction at Joint Hardware and Software Design of Computing Systems

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The main characteristics of schedule construction methods at joint hardware and software design are the following:

- universality of methods in the class of admissible CS architectures;
- compatibility of schedule construction methods and methods of optimal parameters choice of the CS architecture;
- possibility of methods to construct the schedules for CS architectures with various detailed levels.

In the report, different approaches to development of schedule construction algorithms are analyzed. The great area of effective application of greedy algorithms is the construction of schedules at abstract level of solving the task of estimating possibility of CS design and narrowing the considered architecture class. The main area of effective application of algorithms based on a trial and error method (genetic and evolutionary algorithms, simulation annealing algorithms, undirected, directed, and directed with self-training search algorithms) is construction of schedules at a system design level of solving the task of finding architecture's main parameters. The main area of effective application of determined schedule correction algorithms is an improvement of the schedules and details of CS realization at register transfer level.

Combinatorial nature of the tasks, namely the transitivity of order relations, creates a problem of the coordinated modification of variables for deriving the admissible schedule variant on the next iteration. The invariant of the program behavior is introduced as the set of program properties, which should be preserved in all execution variants for their correct operation. The limitations on the schedule are introduced to preserve the program's behavior invariant. For explicit schedule representation (the relations of binding and order are set explicitly), the system of schedule transformation operations is introduced. Its functional integrity is proved. The applicable conditions for operations to preserve an invariant of the program's behavior are obtained.

For parametric schedule representation (the relations of binding and order are set by values of some parameters, by which, with usage of restoring algorithm, these relations can be restored), the possibility of representation of any admissible variant of the schedule and uniqueness of its restoring is proved. The given form of representation is used in genetic and evolutionary algorithms. The parametric representation admits an independent modification of values of all variables in the given intervals, that allows to guarantee deriving the admissible schedule variants at realization of genetic algorithm operations. The given form of representation can be also used in random search algorithms and simulated annealing algorithms.

In the report, the results of numerical research of algorithms will be given.

Rational Decision Making Procedure for a Clinical Task

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The safe delivery problem for women with uterine defect caused by previous Cesarean section is considered. The individual prediction of uterine rupture is impossible because of unknown characteristics of the myometrium. The problem of decision making by physicians concerning the question "continue the preparation to delivery or operate" is formulated and solved in three-stage format. The diagnostic games and methods of modelling medical reasoning are used. The exact formal criteria are stated for every delivery step. The necessary information for decision making is reduced from 450 parameters to 10. The physical model of the 3rd step is built.

Some Algorithms for Multiprocessor Scheduling Problems

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There are n jobs and a finite set of processors. Every job is characterized by a directive interval $[a_i, b_i]$ and the time of performing $0 < \tau_i \leq b_i - a_i$. It is necessary to find a minimal number K of processors for executing all jobs while every job must be performed on a processor continuously (or with interuptions — the second problem). Three binary relations M, F, and N (2-consistent, 1-consistent, and 2-nonconsistent) are introduced on pairs of jobs: $(\alpha_i \leq \beta_j)\&(\alpha_j \leq \beta_i)$, $(\alpha_i \leq \beta_j)\&(\alpha_j > \beta_i)$, and $(\alpha_i > \beta_j)\&(\alpha_j > \beta_i)$. Here $\alpha_i = a_i + \tau_i$ and $\beta_i = b_i - \tau_i$. They correspond to the possibility to perform two jobs on the same processor as follows: in two orders, in one order, and in no order. There are considered six classes of the problem as follows: to use a single relation M, F, or N or the pairs of relations $M \bigcup F$, $N \bigcup F$, or $M \bigcup N$. For all classes, the upper exact estimation proved to be $K \leq \lceil \frac{n}{2} \rceil$.

For classes defined by $M \bigcup F$, $N \bigcup F$, and $M \bigcup N$, the propetries of connectivity, alternating cycles, and cliques are described. It is the sufficient condition for graphs defined by $M \bigcup F$, or $N \bigcup F$ to be interval and for the graph defined by F to be trasitive. As a corrollary, the exact lower estimation of K is proved to be maximal dimension of a clique. The class defined by $M \bigcup F$ is characterized completely as intersections of cliques on M and N. The polynomial algorithms of finding the upper bounds of K for all classes are proposed. The nessesary and sufficient condition for $K = 1$ is found for the problem defined by F. For this problem an algorithm with complexity $O(n^2)$ is proposed. It is known that in general case the problem of checking whether $K = 1$ is NP-complete.

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Computational Technology “Key To Text” for Semantic Search and Retrieval of Textual Information

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The explosive growth of the information resources available electronically created a necessity for efficient semantic search engines. The traditional methods including keyword search and context search are unable to provide semantic filtering in sufficiently large data arrays — the resulting search output is still beyond the scope of human analysis. Similar problems appear in the alternative approach such as a priori semantic indexing, since it requires compilation and standardization of thesauruses. This poses additional difficulties. Also, inefficient filtering puts excessive pressure on the network by tightening the traffic. The efficient analysis and search of the textual information requires profound and sophisticated language and text models and new methods. Such models and algorithms were developed by our research team of the center of New Information Technologies of the Moscow Medical Academy under the grants of the Ministry of Science (N 2.19.2 NCTN-SE, 2D-220/II-94) and Russian Fund for Basic Research (RFBR, N 97-07-131, N 98-01-00929, 00-07-90116).

Our search technology is based on a new original two-level model of understanding and interpreting a text. While the second level requires human interaction for understanding language semantics, the first level is the level, at which purely computational approach offers its help. It turns out that based on combinatorial-statistical analysis, it is possible to synthesize the semantic pattern of a given text, i.e., to generate a small subset of the words, which bears the text's semantics, without referencing to the semantics of the language the text is written in. In particular, the language thesauruses are not used. This phenomenon appears to be true for many European languages (including, for example, Russian and English). The engine based on the two languages has been implemented on a local network of workstations. These procedures have been developed by our team and comprise the core of the intellectual technology that we offer. In other words, the technology delivers a sufficiently detailed semantic (semiotic) portrait of a text and can be used for semantic search, classification, and annotation and development of information resources. While featuring completeness and accuracy of the semantic search and classification, it does not require any form of a priori knowledge of the language besides the language morphology. This facilitates application to any subject area and an extension to more than one language.

The set of developed tools includes:

– computational semantic indexing, classification, and annotation as means of search, analysis, and development of information resources in global communication networks,

– a possibility for a user to describe the subject area by giving text samples,

– multi-language support allowing to use for the text samples a language that differs from the language represented the search results.

The technology is oriented to both end-users and information resource providers and developers.

Physicians' Decision Making Model and Living Knowledge Technology

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We develope and study mathematical models of a physician behavior in solving diagnostic problems as purposeful behavior.

On the base of the formal model, we get the general conditions of feasibility and efficiency of using the inexact knowledge and the unformal interpretation in diagnostic reasoning. We developed the methods of estimating the formalized knowledge efficiency for medical diagnostic problems that are solved in ill-formalized domains. We develope the methodology of constructing the formal language of professional knowledge representation in the case of importance of the both formal and unformal knowledge. The developed structures for the formal representation of knowledge are used as the base for new information technology of knowledge processing, LIVING KNOWLEDGE technology (LKT).

Imagine that you can animate knowledge: you get a possibility to simulate the objects of your professional world in accordance with your or other expert's professional knowledge. By computer application, this wonderful knowledgable world invites you to be an active actor in professional gaming and decision making. The simulated professional world is a good model of real world to train and check youselves. At the same time this world is a powerful tool for knowledge verification in ill-formalized professional fields, where know-how and know-what really mean impersonal and personal knowledge.

By using LIVING KNOWLEDGE technology an expert can create virtual world of his/here intelligence and freely generate scenarios of professional behavior in this knowledgeable world. By using this technology we have developed the set of applied high professional systems for decision support and education in solving diagnostic problems. These systems model clinical cases by using knowledge base. You can have a sort of diagnostic gaming with a modelled patient. In the game, the user creates a scenario based on personal strategies, knowledge, and experience. An intellectual world of LIVING KNOWLEDGE gives you freedom and help.

LIVING KNOWLEDGE invites users to navigate through knowledge space in the ways they do in their professional field and to create the space of their personal knowledge.

Cournot Tâtonnement in Finite Games with Additive Aggregation

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A strategic game Γ is defined by a finite set of players N, and strategy sets X_i and utility functions u_i on $X = \prod_{i \in N} X_i$ for all $i \in N$. Γ is called a game with additive aggregation (an AA game) if each X_i is a subset of reals ($X_i \subseteq I\!R$) and, for each $i \in N$, there exists a function $V_i(s_i, x_i)$ such that $u_i(x) = V_i(\sum_{j \neq i} x_j, x_i)$.

For any AA game and player $i \in N$, we define the best reply correspondence as follows:

$$R_i(s_i) = \text{Argmax}_{x_i \in X_i} V_i(s_i, x_i).$$

In a finite game, $R_i(s_i) \neq \emptyset$ for each admissible s_i automatically; generally, we assume that each X_i is compact and each function V_i is upper semicontinuous in x_i. We also define $r_i(s_i) = \max R_i(s_i)$.

We introduce a couple of binary relations on X ($y, x \in X$, $i \in N$):

$$y \triangleright_i x \iff [y_{-i} = x_{-i} \,\&\, u_i(y) > u_i(x) \,\&\, y_i \in R_i(\sum_{j \neq i} x_j)];$$

$$y \triangleright x \iff \exists i \in N \, [y \triangleright_i x].$$

A Cournot path is a (finite or infinite) sequence $\{x^k\}_{k=0,1,\ldots}$ such that $x^{k+1} \triangleright x^k$ whenever x^{k+1} is defined. A regular Cournot path is a Cournot path such that, for each k, $x^{k+1} \triangleright_i x^k$ implies $x_i^{k+1} = r_i(\sum_{j \neq i} x_j^k)$. A (regular) Cournot cycle is a (regular) Cournot path such that $x^m = x^0$ for some $m > 0$.

For a finite game, the absence of Cournot cycles obviously implies the existence of a Nash equilibrium. It also means that every Cournot path, if continued whenever possible, finds an equilibrium.

An AA game satisfies the strategic complements condition if

$$\text{sign}(V_i(s_i, x_i + \delta) - V_i(s_i, x_i)) \geq \text{sign}(V_i(s_i - \Delta, x_i + \delta) - V_i(s_i - \Delta, x_i))$$

for all $i \in N$ and all admissible s_i, x_i, and $\delta, \Delta \geq 0$.

Theorem 1. No AA game with strategic complements admits a Cournot cycle.

An AA game satisfies the strategic substitutes condition if

$$\operatorname{sign}(V_i(s_i, x_i+\delta) - V_i(s_i, x_i)) \geq \operatorname{sign}(V_i(s_i+\Delta, x_i+\delta) - V_i(s_i+\Delta, x_i))$$

for all $i \in N$ and all admissible s_i, x_i, and $\delta, \Delta \geq 0$.

Theorem 2. No AA game with strategic substitutes admits a Cournot cycle.

Although the formulations of Theorems 1 and 2 are perfectly symmetric, their proofs have literally nothing in common.

A symmetric regular AA game with dual strategic substitutes is an AA game with identical $X_i = \operatorname{Int}[a, b]$, the set of integers satisfying $a \leq x_i \leq b$, and "isomorphic" utilities $u_i(x) = V(\sum_{j \neq i} x_j, x_i)$, where V satisfies the following condition (for all admissible s_i, x_i and $\delta, \Delta \geq 0$):

$$V(s_i, x_i+\delta+\Delta) \geq V(s_i, x_i+\Delta) \Rightarrow V(s_i+\Delta, x_i+\delta) \geq V(s_i+\Delta, x_i).$$

Theorem 3. No symmetric regular AA game with dual strategic substitutes admits a regular Cournot cycle.

It is interesting to note that a Cournot cycle is possible under the conditions of the last theorem. When utilities are the same (and satisfy the dual strategic substitutes condition), a Nash equilibrium exists even if X_i are different integer intervals, but the situation with Cournot cycles is unclear.

Exact analogues of Theorems 1 and 2 are valid for games with separable (not necessarily additive) aggregation. It is unclear at the moment whether Theorem 3 can be generalized in a similar way. Most likely, Theorems 1 and 2 do not need even separability, but no result in this direction has been established. Without any assumption on aggregation, both are definitely wrong.

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Model of Traditional Agricultural Production Sector

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In this work, the interaction of the aristocrat with a traditional sector is considered.

Let us assume a good harvest year with probability q and poor harvest year with probability $1-q$. In a good harvest year the output of the traditional sector is represented by y_0, for a poor harvest year it is represented by y_1. In a good harvest year the traditional sector transfers a part of its produce to the aristocrat of amount x_0, while in a poor harvest year, on the contrary, the traditional sector receives from the aristocrat produce of amount x_1. Thus, the aristocrat undertakes the function of insurance for the traditional sector. In this work, it is supposed that the utility function of the traditional sector is $u(y) = \ln y$. Then the mathematical expectation of this utility function is equal to $\ln(y_0 - x_0)^q(y_1 + x_1)^{1-q}$ or $\ln y_0^q y_1^{1-q}$ in accordance with the availability or absence of insurance correspondingly.

Let us assume that the term of insurance is favorable for the traditional sector, i.e., the inequality $(1 - \frac{x_0}{y_0})^q(1 + \frac{x_1}{y_1})^{1-q} \geq 1$ holds. Let us assume that in a good harvest year the price for produce is represented by p_0 and for a poor harvest year it is represented by p_1. Then the mathematical expectation of the profit of the aristocrat is equal to $qp_0x_0 - (1-q)p_1x_1$. If $p_1y_1 \geq p_0y_0$, then it is not favorable for the traditional sector to interact with the aristocrat. If $p_0y_0 > p_1y_1$, then the solution of the maximization problem of the aristocrat's profit has the following form: $x_0 = y_0[1 - (\frac{p_1y_1}{p_0y_0})^{1-q}]$, $x_1 = y_1[(\frac{p_0y_0}{p_1y_1})^q - 1]$. Thus, the mathematical expectation of the profit of the aristocrat is equal to $qp_0y_0 + (1-q)p_1y_1 - (p_1y_1)^{1-q}(p_0y_0)^q$.

In this work, the problem of efficiency of investments in the traditional sector of agricultural production is investigated. Let us suppose that the investments of volume λ in a traditional sector leads to a produce of $y_0(\lambda)$ in a good harvest year and to a produce of $y_1(\lambda)$ in a poor harvest year. Naturally, it is supposed that $y_0(\lambda)$ and $y_1(\lambda)$ are increasing functions. It is shown that the fulfillment of the following conditions is necessary and sufficient for the aristocrat to be interested in the investments: $\frac{dy_0(\lambda)}{d\lambda}/\frac{dy_1(\lambda)}{d\lambda} > (1-q)p_1[(\frac{p_0y_0(\lambda)}{p_1y_1(\lambda)})^q - 1]/\{qp_0[1 - (\frac{p_1y_1(\lambda)}{p_0y_0(\lambda)})^{1-q}]\}$.

The Complexity of the Computation of Maximin for Square Function

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We represent polynomial algorithm to solve the following problem:

$$\max_{x\in B}\min_{y\in C}\left(xAy^t + ax^t + by^t\right).$$

Here $x \in R^n, a \in R^n, y \in R^m, b \in R^m$; B and C are polytopes of dimensions n and m; and A is a $(n \times m)$-matrix.

We prove NP-hardness of the problem whith B and C being coherent polytopes. We reduce the NP-complete 3-feasibility problem to it.

We reduce the special case of cyclical games to the initial problem.

Hedge Portfolio for Option Pricing with Different Share Prices for Selling and Buying

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The classic problem of hedging portfolio for European "call" options is considered. At time moments $n = 0, ..., N$ the market is characterized by the value of soft money B_n and share value S_n. The market evolution is defined with the parameter r that determines the increase (or decrease) of the value of money as $B_n = B_0(1+r)^n$, and a and b $(b > a)$ describing the share price changing: $S_n = (1+\rho_n)S_{n-1}$. Here ρ_n is b or a, corresponding to the share price rising or decreasing at the given moment. The option of price $C(N)$ is released. At moment N the releaser must be ready to sell the share to the option holder at price K if the holder wishes. The releaser should distribute his money in such a way that market changes would not ruin him. That means that his final capital X_N is not less than $\max\{S_N - K, 0\} \equiv (S_N - K)^+$. His portfolio is $\pi_n = (\beta_n, \gamma_n)$ $(n = 0, 1, \ldots, N)$ with the capital $X_n = \beta_n B_n + \gamma_n S_n$, where β_n is the amount of money and γ_n is the amount of the shares at moment n.

Denote $\Delta\beta_n = \beta_n - \beta_{n-1}$; $\Delta\gamma_n = \gamma_n - \gamma_{n-1}$. *Self-financed portfolios* are studied, such that $\Delta\beta_n B_{n-1} + \Delta\gamma_n S_{n-1} = 0$ $\forall n = 1, ..., N$. The portfolio with the minimal starting capital is called *a hedge*. It has been shown that if $-1 < a < r < b$ then this portfolio exists, and its components can be calculated [1].

We introduce the following generalization: at each moment n the share can be sold at price S_n and bought at S'_n, where $S'_n \geq S_n$: $S'_n = (1+\omega)S_n$ $(\omega \geq 0)$. This model differs significantly from the classic one. First, in the classic case, it makes no difference how the starting portfolio is built since converting money to shares can be accomplished without any losses. Also it changes the requirements at moment N, because the portfolio components should be converted into one share, which is not the same as having capital $(S_N - K)^+$. Secondly, in the classic model, at any moment the portfolio is determined by its capital. The components β_n, γ_n are determined by X_n. In our model, it is not so due to the change of the self-finance condition. Separate equations should be solved to find both β_n and γ_n. All that leads to a system of two-parametric recurrent equations. It can be solved, though the analytical formulas appear inconvenient for numerical calculations. Nevertheless, an efficient way for numerical calculations has also been developed.

The comparison of the solutions of the classic and presented models was carried out, and some inequalities for the portfolio components were deduced. At last, these results were applied to determine the behavior of a firm that meets different rates of exchange.

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Error Bounds for the Aggregation in the Generalized Transportation Problem

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Aggregation/disaggregation is used to reduce analysis of a large generalized transportation problem (GTP) to a smaller one. Bounds for the actual difference between the aggregated objective and the original optimal value are used to quantify the error due to aggregation and estimate the quality of the aggregation. Two types of the error bounds are considered, a priori error bounds and a posteriori error bounds. A priori error bounds may be calculated after the model has been aggregated, but before the smaller aggregated model has been optimized. A posteriori error bounds are calculated after the aggregated model has been optimized. In both cases the bounds should be calculated without optimizing the original detailed model. Error bounds allow the modeler to compare various types of aggregation and, if the error bounds are not adequate, modify the aggregation strategy in an attempt to tighten the error bounds.

There are two principal questions associated with the error bounds. The first one is addressed to the quantitative quality of the bound, i.e., the difference of an error bound from the actual error. The second question is how to compare different aggregation strategies based on the error bound. It is not obvious that the aggregation yielding the tightest error bound has the smallest actual error. Moreover, different bound calculation techniques may result in different correspondence between the error bound and the actual error.

We focus on the second question and study customers' aggregation in the GTP. We derive a family of a priori and a posteriori error bounds. A computational experiment is designed to

a) study the correlation between the bounds and the actual error and

b) quantify the difference of the error bound from the actual error.

The experiment shows significant correlation between the a priori bound and the actual error: the a priori bound varies in the same way the actual error does. That is, if a number of different aggregated models is considered for the same original GTP, the smallest actual error corresponds to the smallest a priori error bound. At the same time numerical value of the a priori bound can differ significantly from the actual error, while the a posteriori bound is much tighter than the a priori one. These preliminary results indicate that calculating the a priori error bound is a useful strategy to select the appropriate aggregation level. After the aggregated problem has been selected and optimized, the a posteriori bound provides a good quantitative measure for the error due to aggregation in the GTP.

Optimization of a Guarantee for a Class of Differential Games

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The report is devoted to sufficient conditions of existence of the solution for a class of nonlinear differential games. Also we consider ways of finding the solution in a class of positional strategies and demonstrate results of computing test examples. The following class of two-person differential games is studied. In Euclidean n-dimensional space, $n = n_1 + n_2$, there is a motion of the phase vector $x = (x_1, x_2)$, $x_1 \in R^{n_1}$, $x_2 \in R^{n_2}$, satisfying the equations

$$\dot{x}_1 = f_1(x_1, x_2), \tag{1}$$

$$\dot{x}_2 = f_{21}(x_1, x_2, v) + f_{22}(x_1, x_2)u, \tag{2}$$

where $u \in R^{n_1}$ is the control vector, $u \in P$, $v \in R^{n_3}$ is the vector of uncertainty, $v \in Q$, P and Q are convex sets, $0 \leq n_3 \leq n_2$. The conditions of existence, uniqueness, and unlocal continuability of the solution of the system (1)–(2) are imposed on the functions f_1, f_{21}, and f_{22} for Lebesgue-measurable controls $u(t) \in P$ and uncertainty $v(t) \in Q$. On a part of phase variables the condition

$$f_3(x_1) = 1 \tag{3}$$

is imposed. The initial and terminal conditions are given by

$$x_{10}, x_{20} \quad (f_3(x_{10}) = 1) \tag{4}$$

and

$$x_{1*}, x_{2*} \quad (f_3(x_{1*}) = 1). \tag{5}$$

The quality of a control is characterized by the following functional:

$$J = \int_0^T (u, Du)dt, \tag{6}$$

where D is a positively definite matrix $(n_1 \times n_1)$. We consider the problem of existence of control $u(t)$ in a class of positional strategies, guaranteeing the following estimation:

$$\max_{v \in Q} \min_{u \in P} J(u(\cdot)) \leq \gamma. \tag{7}$$

Here γ is a positive constant.

The report consists of three parts. In the first part the sufficient conditions are given for existence of the solution of the problem (1)–(7) in classes of counterstrategies and positional controls. The ideas of a method of control with help of the model [1-4] and the method of dynamic regularization [2] are applied. In the second part there are suggested algorithms of calculating controls for all the investigated classes of strategies. In the third part the results of computations are given for control of a solid body orientation at the presence of uncertain disturbers.

References

1. Pontriagin, L.S. The selected works. V.2. M.: Nauka. 1988.

2. Osipov, Yu.S., Vasiliev, F.P., Potapov, M.M. Foundations of a method of dynamic regularization. MSU. 1999.

3. Krasovsky, N.N., Subbotin, A.I. Position differential games. M.: Nauka. 1974.

4. Nikolsky, M.S. A minimax problem of control // Works of the mathematical institute of RAS. V.185. 1988. P.187–192.

OR in Parameter Estimation of Discrete Distribution

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OR methods create great posibilities for solving classical statistical problems. A lot of statisical problems has large dimension and many parameters, which should be described and worked up. Enlarging a set of permissible decisions by including randomized decision functions make the problem much more complicate.

In 1990 at PSTU was created a minimax estimation computer system (MECS). It can solve estimation problems for different payoff functions. Thus, point and interval estimation problems and hyphothesis tests are solved within a common approach.

A statistical game Γ_P is generated by a family $P = \langle X, P_\theta \rangle_{\theta \in \Theta}$ of probability distributions P_θ on a set X and a strategic game $\Gamma = \langle \Theta, D, w \rangle$, in which D is a set of permissible Statistician actions or decisions and w is the loss function given on the direct product $\Theta \times D$.

In statistical game $\Gamma_P = \left\langle \Theta, D^X, R \right\rangle$, Nature selects a parameter value $\theta \in \Theta$ unknown to Statistician. Observing a value x of a random variable X_θ with distribution P_θ, Statistician fulfills an act d from the set D. Statistician's losses result in $w(\theta, d)$. In statistical game Γ_P, the risk function R is defined by the expectation of Statistician's losses, that is, $R(\theta, \mathbf{d}) = \int w(\theta, d(x)) dP_\theta(x)$. The function $\mathbf{d} = d(x)$: $X \to D$ and, thus, $\mathbf{d} \in D^X$, is the decision function of Statistician.

It is sometimes convenient to assume that the set of permissible actions of Statistician $D_x, x \in X$, varies together with outcomes of observation of a random variable X. In this case, a statistical game $\Gamma_P = \langle \Theta, D, R \rangle$ is generated by a family $P = \langle X, P_\theta \rangle_{\theta \in \Theta}$ and by a family of strategic games $\Gamma_x = \langle \Theta, D_x, w_x \rangle$, $x \in X$. Then the set of decision functions of Statistician $D = \prod D_x$ consists of functions $d(x) : X \to D_x$, and the loss functions w_x vary together with x. This definition of statistical game differs from the traditional one, it is especially convenient for the problems of an interval parameter estimation.

We found the least number of observation for probability of success estimation in Bernulli trials when precision and reliability are fixed. A decision function is constructed, using intervals with fixed width. The number of observation is reduced for 15-30%. The problem of tolerance estimation or prediction of a value of random variable based on a single observation of another random variable also can be solved by the MECS.

Let us assume as given two independent families $\{X_\theta\}, \{Y_\theta\}$ of random variables and corresponding families $P_X = \langle X, P_\theta \rangle_{\theta \in \Theta}$, $P_Y = \langle Y, Q_\theta \rangle_{\theta \in \Theta}$, with a common parameter set Θ. The suite of intervals $d = \{d(x)\}, d(x) \subseteq Y$, is designated a suite of (β, γ)-tolerance intervals for random variables X_θ, if the proportion of a random variable Y_θ, falling in an interval $d(X_\theta)$, is not less than β and the probability of such event is not less than γ for all $\theta \in \Theta$, that is $P(P(Y_\theta \in d(X_\theta)) \geq \beta) \geq \gamma$.

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Reference

1. Lutsenko, M., Ivanov, M. Minimax confidence intervals for the parameter of hypergeometric distribution, Avtomatika i telemechanika, 2000, 7, pp. 68-76.

Minimax Problem of Control of a Parabolic Variational Inequality

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The minimax control problem in the form developed by Ekaterinburg school and based mostly on the works by N.N. Krasovskii consists of two problems. These problems are represented for two antagonistic players in a two-person game. The players realize the process of control of a dynamic system by means of different inputs. One of them (a partner) chooses a control in the class of feedback controls (positional strategies), the other (an opponent) does not restrict himself in the choice of a control. As a rule, the quality of system's trajectories is estimated by a cost functional. The task of the partner is to choose a positional strategy which minimizes the maximal value of the cost functional. The task of the opponent is opposite: to choose a control which maximizes the minimal value of the cost functional.

The theory of guaranteed control problems (they are also called positional differential games) for systems described by ordinary differential equations has been presented in details in the monographs [1, 2]. The foundation of the theory of guaranteed control for some classes of distributed systems in the framework of the aforesaid formalization has been laid in the works by Yu.S.Osipov and his successors [3–6]. In the present report, the problem of minimax control is investigated for the new class of distributed systems, namely, for parabolic variational inequalities. Under the so-called condition of "entire domination" the minimax positional strategy of the partner and the maximin positional strategy of the opponent are constructed. The theorem of existence of a saddle point is established.

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References

1. Krasovskii, N.N., Subbotin, A.I. Game-theoretical control problems. Berlin: Springer, 1988.
2. Krasovskii, N.N. Controlling of a dynamical system. Moscow: Nauka, 1985 (in Russian).
3. Osipov, Ju.S. Differential games for systems with hereditary // Dokl. AN USSR, 1971. Vol. 196, no. 4, pp. 779–782 (in Russian).
4. Osipov, Ju.S. A positional control in parabolic systems // Prikl. math. mechan., 1977. Vol. 41, no. 2, pp. 23–27 (in Russian).

5. Osipov, Ju.S., Ohezin, S.P. On the theory of differential games in parabolic systems // Dokl. AN USSR, 1976. Vol. 226, no. 6, pp. 1314–1317 (in Russian).
6. Maksimov, V.I. A problem of positional control in nonlinear parabolic systems // Different. Equat., 1992. Vol. 28, no. 6, pp. 961–967 (in Russian).

Mathematical Methods for Maximizing Survivability of Multiuser Systems with Network Structure

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The notion of normative survivability of multiuser network systems is formalized. The classes of hierarchical and ring network systems are studied from the point of view to provide normative survivability. Both normative and muticriterial survivability of three-commodity networks are investigated and compared in the paper.

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Nondominated Solutions of Noncooperative Differential Games

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In this study, Nash equilibrium in the class of positional strategies is investigated for the nonantagonistic two-person differential game with terminal payoff functions.

It was shown by A.F. Kononenko that every Nash equilibrium in the nonantagonistic two-person positional differential game is connected with a certain motion of dynamic control system. In particular, the value functions of auxiliary antagonistic positional differential games satisfy some inequalities along this motion.

Consequently, there exists infinite number of equilibrium motions in the class of positional strategies. Therefore, it is very important to determine nondominated and Pareto optimal equilibria.

The convexity of the payoff functions of both players and the set of solutions of the dynamic system are supposed.

Results obtained in this study show that Pareto optimal equilibria are connected with such equilibrium motions that the value functions of auxiliary antagonistic games satisfy certain conditions of monotonicity. The suggested method allows to obtain more effective conditions for the equilibrium of the certain positional differential game with terminal payoff functions to be Pareto optimal. For example, these conditions are presented for a class of linear games and games with simple motions.

More on the Sequential Generalized Lorenz Dominance

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In 1987 Atkinson and Bourguignon introduced the sequential generalized Lorenz (SGL) dominance for comparing heterogeneous income distributions which can be naturally decomposed into homogeneous sub-groups; i.e., groups of people having the same "non-income characteristics". This partial ordering, now widely used among practitioners and theorists in the area of inequality measurement, was shown to be equivalent to welfare dominance according to a class of utilitarian SWFs satisfying certain desirable properties. This equivalence establishes the normative relevance of the SGL dominance. Ok and Lambert (1999) show that this result also holds for SWFs which are not additive it between types. In this paper we further extend the Atkinson and Bourguignon equivalence by relaxing the additivity assumption *both within and between types*, thereby identifying the largest class of SWFs consistent with the SGL partial ordering.

Fast Method for Finding an Optimal Architecture for a Neural Network Approximator

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In this work, a problem of finding an optimal structure for a neural network approximator is considered. The problem is stated as follows: for a given finite set X of evaluations of an unknown function F, build an approximation of F with a feedforward neural network of minimal complexity, satisfying specified constraints on a quality of approximation.

Quality of approximation of F by neural network NET is determined by a mean-square error (MSE) of approximation, calculated at the points of set X. Any neural network can be viewed as a pair (A,W), where A is an architecture of a network (determined, in most simple case, by a number of hidden layers, number of neurons in each layer, and types of activation functions), and W is a combined vector of all weights (adjustable parameters) of the network. Complexity C(NET)=C(A) of a network NET=(A,W) is an application-defined real-valued function of a network architecture. Examples of such function are as follows: total number of network weights, neurons, or the number of operations required to calculate an output signal of a network.

The problem of finding an optimal network can be formulated as a mixed integer constrained optimization problem: find such (A,W) that minimizes C(A) subject to the constraint that value of MSE(A,W) must not be greater than some predefined value (necessary quality of approximation).

A common way to solve the problem is to use a search procedure (stochastic or deterministic) in the space of possible network architectures A, and evaluate (via training a network) best MSE, achievable for each architecture. Serious disadvantage is that multiply network training sessions are required to find a solution, which results in a great computational complexity.

A modification of this approach, aimed at reducing computational complexity of finding an optimal architecture, is presented in this work. Main idea of presented modification can be formulated as follows: the more similar are network architectures, the more similar quality of approximation these networks can achieve after training. If this supposition is true, it is possible to estimate best MSE, achievable for a network with a given architecture, without training, but only by considering known values of MSE, achieved for a networks with architectures similar to a given one, if such information exists. Using such estimation of best MSE, it is possible to reduce a number of network training sessions required to find an optimal network, replacing them with estimations of best MSE, which are based on the results of training sessions that already were run.

In order to implement such method for best MSE estimation, some distance metric in a space of network architectures is required (to measure the similarity of architectures). One of such metrics is presented in this work, along with some details of MSE estimation method and results of experiments.

House-Selling Problem in Game-Theoretic Way

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In this work, we investigate the of problem of sequential choice in which the player (a company) owns m objects and sets a price for each of them. The offers to buy an object come sequentially one by one during certain periods of time. Every offer has its "size" reprezented by non-negative random value X. If the offer X is not lower than price s then the player sells one of the objects for the price s, otherwise the offer is rejected and the buyer comes to another firm. When the trades are finished the player gets a fixed quantity R for every object that has not been solved. Our problem is to find optimal strategies in various cases where other players exist.

Decomposition of Linear Programs with Binding Constraints and Variables

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It is known that to decompose the aforesaid problems we can use two Dantzig-Wolfe processes. One of them should be included into internal process of the other. It is also known that for degenerate problems when the crossing block is zero these processes can be performed parallelly. However, it can be shown that this condition is not important, because there always exists a transformation, which reduces nondegenerate problem with special structure of constraints to degenerate one. Mathematical properties of this case are presented.

Models for Market Risk Measurement

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The aim of this paper is to analyze the applicability of Value-at-Risk measure to unstable financial markets. Two kinds of models are considered: normal (variance-covariance) model and extremal events model. Normal model was implemented via both parametric and nonparametric methods of covariance matrix estimation: constant- and exponentially weighted covariances, GARCH-models, and wavelet-based nonparametric model. A number of tests were performed for backtesting the models. Two main criteria were considered: 1) precision and 2) efficiency. Model precision is responsible for correspondence with VaR definition, while efficiency characterizes correlation of VaR and real profit/losses. The results obtained point that traditional parametric models can be ill suited for unstable markets, while the proposed nonparametric model shows good results even on those markets. Moreover, multicriteria analysis of efficiency shows Pareto-domination of nonparametric wavelet-based model.

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Informational Effectiveness of Double Auction

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During the last decades, more and more attention is being paid to a special kind of trading system that is usually referred to as the continuous double auction. Now in almost every country there are a number of trading systems and stock exchanges where people can sell and buy various goods and financial tools. The largest stock exchange is the NYSE (New York Stock Exchange) and the largest trading system is the NASDAQ (National Association of Securities Dealers Automated Quotations). Some properties of the double auction are considered in this study. As a base model of the market, a simple stationary stochastic market model is taken. In this model all traders put random bids and at random moments in time. It is shown that the main principle of the double auction, the competition of bids and asks, accounts for prices to reach a stationary equilibrium and that the presence of the queue in the trading system greatly reduces price oscillations. Moreover, an explicit dependence is obtained for the market stabilization time based on the queue depth. The second part of the study is the analysis of the market capability to reveal private information available to all traders. The analysis is based on the stochastic messaging model where every trader receives information about the real stock price with some random error. This information is private which means that different traders receive different information and they do not reveal the knowledge to other traders. It is shown that if traders obey to the rules of the double auction it is possible to obtain additional information about the real stock price by analyzing current market prices. This 'smart' trader behavior can, under certain conditions, result in faster stabilization of prices. The analysis of the double auction is to be considered in the boundaries of development on the rational expectations theory.

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Evolution of the Securities Market in Russia

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This report analyzes in details stages of creation, development, and crisis of the securities market in Russia on the basis of large amount of numerical information.

In the 90s, the modern securities market infrastructure was created in Russia. With its help it is possible to redistribute the long-term investment for the purpose of real sector modernization. The first experience of estimating and transferring the property rights, resolving conflicts of interests, and attracting money resources for a company's development has been gained in the securities market.

Up to the autumn of 1997 the securities market had quickly increased its operational capacity. As the result of the crisis of 1998 the market was physically destroyed. In the report, characteristics of basic financial tools in comparison with other countries are considered.

Twice in the past (October 1997 and May 1998) the Russian securities market served as a trigger mechanism for the unleashing of crises in all the sectors of the national financial market. Why is the true picture of the financial crisis of 1997-98 so important? What is in it that a financial regulator (the Federal Securities Commission of Russia) but not the Ministry of Finance of Russia or the Bank of Russia takes a part of the responsibility for inflation, absence of the investment, and decrease of the living standards? The danger is that if, once created the share market that is one of the most risky and volatile, the state and the professional community do not try to understand and recognize the problems, which have confronted the market, then the Russian market of shares will not be restored on a new, safer basis.

The securities market, which is being reconstructed in 1999-2000 is still insignificant in terms of volume. Speculative investor's majority does not provide an opportunity to attract large means in the capital market for financial investment.

Nobody among the professionals doubts that the Russian shares market is manipulative. We will never learn if the spasm of the Russian shares market, its headlong rise and fall in 1996-1998, was a manipulative pool or the manipulation carried out by several players. We will never learn if it really was a manipulation. We can only suspect that if we take two groups of investors, a foreign and a domestic one (including off-shore), and try to understand the final account of wins and losses in the aftermath of 1996-98, then there will be a consolidated group of foreign investors in the final gain.

After the "Russian Bubble" the view of securities as being a "fictitious capital", a "gamble" has prevailed, first of all, in the sphere of the state apparatus. It is very important that at the time of coming ideological turns, volte-face in the state authority, the share market would be able to demonstrate the ability to create projects generating cash flows in the real sector, as the long-term investments.

To the Theory of Kernels and Hulls in Transportation Polyhedrons

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A method to approximate matrices consisting of zeros and unities with extremal matrices is proposed. For such matrices which belong to a common transportation polyhedron two special matrices are built. They are referred to as kernel and hull. The kernel and the hull are the best approximation in the class of extremal matrices.

Hierarchical Repeated Game with Restriction on the Time of Observation

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The two-person game is considered. The first player has the right of the first move. He has the possibility to receive information in both discrete and continuous way. The fixed time is spent on the separate observation. The limitation exists on the total time of observation for the behaviour of the second player. In the common case the total time is less than the duration of the game. It is clarified whether the first player is able to receive the same gain as he receives in the game with the continuous observation during all the game. The optimal regimes of the reception of information are defined.

Kyoto Index for the Power Plants in the USA

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The paper deals with an application of multi-criteria risk assessment towards some environmental and economical aspects of electrical energy production, supply, and distribution. The aspects are described by means of a set of parameters or indicators, such as Demand Side Management (DSM) Program at the utility (DSM index = Energy saved /Total Energy sold), Renewable energy ratio (Energy generated using Renewable resources/ Total Energy produced), Energy Efficiency (amount of CO2/ Kw/h) related to greenhouse gases and global warming, Human health effects (NOx, SO2, suspended particles, mercury and other toxic substances, quantity and type of hazardous waste generated during energy production), Ecological effects (mining for fuels that are used in electric energy generation, heating of environment, NOx, SO2, particulates, mercury and other toxics, quantity and type of hazardous waste generated during energy production), Acid rain impact on lakes, other ecosystems and cultural heritage, Catastrophic potential (e.g., nuclear power plant or large dams), etc.

The related multi-criteria optimization problem is posed aiming at minimizing risks of hazardous impact of the power plants on their natural environment and involved human population and staff. Kyoto Index is introduced as a tool to evaluate electric energy producers, which will promote energy conservation, energy efficiency, renewable energy production, and minimal impact on the environment. Kyoto Index is an integral indicator for the expected risk of the hazards. Kyoto Index is intended to evaluate electricity producers in deregulated area of USA and choose the top 10-25% of those companies that offer reasonable cost/unit electricity. It has been tested on the data related to the power plants of Ohio, USA.

Relevant software has been developed to implement Kyoto Index for Microsoft Excel spreadsheet. Three different algorithms have been used. One of the algorithms is based on the expert estimates for the relevant parameters. Kyoto Index is evaluated for each power plant as a weighted sum of the parameters for the plant, the weights being assigned by the experts. The other algorithms are oriented to real databases on the power plants and utilities of the USA. The data forms a data matrix with rows related to the plants and the columns related to the parameters. The initial data matrix parameters are measured in ordinal scales. The matrix is transformed so that the maximal value of each parameter would correspond to a maximal risk of a hazard for nature and/or people. Singular value decomposition of the data matrix is used to evaluate integral indicator (Kyoto Index). The third algorithm is based on Pareto clustering technique for the same data matrix.

The study has been initiated and sponsored by GAIA UNLIMITED, Inc. Methods and techniques used to develop Kyoto Index have been developed with support of the Russian Foundation for Basic Research (Grant RFBR 00-01-0197). Authors are greatly indebted to Prof. Alexander Lotov (Computing Center of RAS), Ned Ford (Cincinnati, Ohio), and Congressman Dennis Kucinich (USA) for valuable discussions.

On the Optimal Organization of the Tax Service

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The present paper considers a game-theoretic model related to tax evasion and corruption in the tax system. Suppose that the government can employ tax inspectors of two types: "honest" auditor or "dishonest" auditor. The distinction of these types is expressed in different costs of audit. "Honest" auditor costs more than the second type of auditor. That's why the government has three strategies to hire auditors.

Variant 1. Hiring only "honest" inspectors. In this case, there is no need to review an audit.

Variant 2. The strategy of hiring both types of "honest" and "dishonest" inspectors. Here "dishonest" inspectors audit taxpayers and "honest" inspectors review some audits. In this case, the government has to take into account an opportunity that a detected tax evader will bribe the "dishonest" inspector who discovered the fact of tax evasion.

Variant 3. Hiring only "dishonest" inspectors. In this case, the government will not review the work of inspectors. The only possibility to exclude bribing in this case is to establish the probability of auditing so high that a taxpayer will not evade even if there is a possibility to bribe a tax inspector.

Variant 1 was studied in [1]. Variants 2 and 3 were partially analyzed in [2]. Our purpose is to complete the analysis and to determine the optimal variant of audit organization depending on parameters of the model.

Assume that there are only two possible levels of income ($I_L < I_H$), obtained with probabilities $1-q$ and q, respectively. The low income is free of tax and the tax on the high income is T. Thus, taxpayers with high income may have an incentive to report I_L. If a taxpayer reports low income, the tax agency may wish to conduct an audit. An audit costs c for a "dishonest" inspector and $\tilde{c}$ for an "honest" inspector, $c < \tilde{c}$. It always reveals the true income. The fine for under-reporting is F, and it includes the original tax liability.

Consider variant 2. The tax authority can send another (honest) auditors if the first audit confirms the low income. The authority penalizes the first auditor if the review reveals that he has concealed tax evasion. The penalty in this case is equivalent to a monetary fine of value $\tilde{F}$. The probabilities p and p_H of auditing and reviewing, respectively, are established by the authority. The tax authority aims to maximize net tax revenue, including taxes and fines minus all audit costs.

We start by discussing the bargaining over the amount of the bribe b, in the case where a tax evader has been caught by an audit. Bribing is profitable for the taxpayer and the auditor if $b + p_H F < F$ and $b > p_H \tilde{F}$, respectively. Let denote $\hat{p} = \frac{T}{F}$, $\hat{p}_H = \frac{F}{F+\tilde{F}}$.

According to the results in [2], mutually benefited collusion between a taxpayer and a dishonest inspector is possible if $p_H < \hat{p}_H$. If this inequality holds, let $b = \gamma F(1-p_H) + (1-\gamma)p_H \tilde{F}, \gamma \in (0,1)$. A taxpayer with high income will cheat if $p(b + p_H) < T$.

Let $\hat{p} > \gamma$. In this case, for variants 1, 2, 3 the following expressions for the income of the government were received.

Variant 1: $R_1^* = qT - \frac{T}{F}(1-q)\tilde{c}$, $p^* = \frac{T}{F}$, if $R_1^* > 0$.

Variant 2. If $\hat{p}_H \leq \dfrac{c(1-\gamma)}{\gamma\tilde{c}}$ then $R_2^* = qT - (1-q)\dfrac{T}{F}(c + \tilde{c}\dfrac{F}{(F+\tilde{F})})$, $p^* = T/F$, $p_H^* = F/(F+\tilde{F})$.

If $\hat{p}_H > \frac{c(1-\gamma)}{\gamma\tilde{c}}$ then $R_2^* = qT - (1-q)(c + \tilde{c}\dfrac{T-\gamma F}{(1-\gamma)(F+\tilde{F})})$,
$p^* = 1,\ \ p_H^* = \dfrac{T-\gamma F}{(1-\gamma)(F+\tilde{F})}$.

Variant 3: $R_3^* = 0$ because it is profitable to evade from the tax even under $p = 1$.

Theorem. Let $\hat{p}_H < 1-\gamma$, $\frac{c}{\tilde{c}} \geq \frac{\tilde{F}}{F+\tilde{F}}$ or $p_H \geq 1-\gamma, \frac{c}{\tilde{c}} \geq \hat{p} - \hat{p}_H\frac{\hat{p}-\gamma}{1-\gamma}$. Then hiring of "honest" auditors is optimal if $qF > (1-q)\tilde{c}$. In this case the optimal audit probability is $\hat{p}$ and the net tax revenue is $R^* = R_1^* = qT - \frac{T}{F}(1-q)\tilde{c}$. Otherwise there is no need for auditing this group of taxpayers: $p^* = 0$, $R^* = 0$.

Let $\hat{p}_H < 1-\gamma$, $\frac{\gamma}{1-\gamma}\frac{F}{F+\tilde{F}} < \frac{c}{\tilde{c}} < \frac{\tilde{F}}{F+\tilde{F}}$. Then variant 2 provides the maximal tax revenue if $qF > (1-q)(c + \tilde{c}\hat{p}_H)$. In this case, the optimal audit probability is $\hat{p}$ and optimal probability of reviewing is $\hat{p}_H$. The net revenue is $R^* = R_2^* = qT - (1-q)\frac{T}{F}(c + \tilde{c}\frac{F}{(F+\tilde{F})})$. Otherwise there is no need for auditing this group of taxpayers: $p^* = 0$, $p_H^* = 0$, $R^* = 0$.

Let $\hat{p}_H < 1-\gamma$, $\frac{c}{\tilde{c}} \leq \frac{\gamma}{1-\gamma}\frac{F}{F+\tilde{F}}$ or $\hat{p}_H \geq 1-\gamma$, $\frac{c}{\tilde{c}} < \hat{p} - \hat{p}_H\frac{\hat{p}-\gamma}{1-\gamma}$. Then variant 2 provides the maximal tax revenue if $qT > (1-q)(c + \tilde{c}\frac{T-\gamma F}{(1-\gamma)(F+\tilde{F})})$. In this case, optimal probabilities are $p^* = 1$, $p_H^* = \frac{T-\gamma F}{(1-\gamma)(F+\tilde{F})}$ and the net tax revenue is $R^* = R_2^* = qT - (1-q)(c + \tilde{c}\frac{T-\gamma F}{(1-\gamma)(F+\tilde{F})})$. Otherwise there is no need for auditing this group of taxpayers: $p^* = 0$, $p_H^* = 0$, $R^* = 0$.

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References

1. Reinganum, J.R, Wild, L.L. Income tax compliance in a principal-agent framework // Journal of public Economic, 1993. V.50, 345-369.

2. Vasin, A, Panova, E. Tax collection and corruption in fiscal bodies // The Economics Education and consortium, 2000, No 99/10.

Branch-and-Bound Method for the Network Vulnerability Problem

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In the paper, the problem of multicommodity network vulnerability [1] is considered. We suggest the Simple Cutsets algorithm for solving this problem. The algorithm is based on branch-and-bound method. The main idea of Simple Cutsets algorithm is to use combinations of simple cutsets and edges as branches for seaching for an optimal solution. We are interested in testing this variant of branch-and-bound method for different network vulnerability problems.

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Reference

1. Nazarova, I.A. The Problem of Vulnerability for Telecommunication Networks // Abstracts of The 2nd Moscow International Conference On Operations Resaerch (Moscow, November 17-20, 1998). Computing Center of RAS. Moscow, 1998.

About Flexibility of Linear and Nonlinear Control Systems

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Consider a control system (see, for example, [1])

$$\dot{x} = f(x, u), \quad x(0) = x_0, \tag{1}$$

where $x \in R^n$, $u \in R^p$, $t \in \Delta = [0, T]$. Controls $u = u(t)$, $t \in \Delta$, are measured and bounded in absolute value vector functions. Solutions $x(t)$ of the Cauchy problem (1) under $u = u(t)$ are considered in the class of absolutely continuous vector functions on Δ. We are interested in behavior of the following outputs of the system (1):

$$y(t, u(\cdot)) = \pi x(t, u(\cdot)), \quad t \in \Delta, \tag{2}$$

where π is a $(m \times n)$-matrix. We can consider the outputs (2) as the elements of various Banach spaces. Let one of such Banach spaces, the space B, be fixed.

Definition. The control system (1),(2) is called the flexible system with respect to the space B if the outputs (2) are dense in B (see [2]).

The conception of flexibility generalizes the Kalman's conception of controllability (see, for example, [1]) on a functional level. We have obtained certain constructive sufficient conditions for flexibility in linear and nonlinear cases. This report continues and developes the results of our papers [3] – [5].

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References

1. E.B. Lee, L. Markus. Foundations of optimal control theory. Moscow: Nauka, 1972 (in Russian).
2. A.N. Kolmogorov, S.V. Fomin. Elements of the theory of functions and functional analysis. Moscow: Nauka, 1989 (in Russian).
3. M.S. Nikolskii. Certain problems of tracking a given motion with noise // Differencialnye uravn, 31, N. 11, 1995, pp. 1866–1873 (in Russian).
4. M.S. Nikolskii. About one problem of tracking a given motion. Flexible systems // Doklady Akad. Nauk USSR, 350, No 6, 1996, pp. 739 - 741 (in Russian).
5. M.S. Nikolskii. Nonlinear flexible control systems // Izvestia Akademii Nauk, Teoriya i sistemy upravleniya, No 4, 1996, pp. 5-7 (in Russian).

Two Models for Multicriterial Games

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In multicriterial decision making all the criteria are important, but the other model is considered in multicriterial antagonistic games (MAGs). It corresponds to the multicriterial case where an arbitrary criterion (may be the single one) is to be maximized. This nonstandart formulation is also multicriterial since we have not fixed the maximized criterion beforehand. The model appears in cooperative games without side-payments [1]. The MAG Player I is a coalition. Her vector payoff function $\Phi = (\varphi_1, \dots, \varphi_Q)$ is the vector of payoffs of the coalition members. The MAG Player II assembles the rest players with the aim to disorganize the coalition, i.e., to make it unprofitable at least for a single member. Evidently, she wants to minimize Φ. However, her payoff function is not $-\Phi$ because she is not interested in all φ_i.

Opposite to MAG, a multicriterial zero-sum game (MZSG) is a two-person game with vector payoff function of Player II equal to minus vector payoff function of Player I. Thus, the both players of MZSG are multicriterial decision makers in a common sense. In order to differ MAG from MZSG, we consider two types of vector estimates: *guaranteed* and *weak*. The first type gives the following set of estimates for vector $\Phi(v)$ while minimizing: $\{\psi \in \mathbb{R}^Q | \psi \geq \Phi(v)\}$. The set of weak estimates for the vector $\Phi(v)$ is $\{\psi \in \mathbb{R}^Q | \psi \not< \Phi(v)\}$, where a crossed inequality means negation of the vector inequality. We believe that Player II of MZSG uses guaranteed estimates, but the MAG Player II uses weak estimates. Of course, in the singlecriterial case guaranteed and weak estimates are equal.

Player I uses guaranteed estimates in both multicriterial games. Her multicriterial minimax and maximin are equal to

$$\mathop{\mathrm{Min}}_{y\in Y}\overline{\mathop{\mathrm{Max}}_{x\in X}}\Phi(x,y) = \mathcal{F}_{\leq} \stackrel{\text{def}}{=} \mathrm{Max}\bigcap_{y\in Y}\bigcup_{x\in X}\{\psi\in \mathbb{R}^Q|\psi\leq\Phi(x,y)\} \quad \text{and}$$

$$\overline{\mathop{\mathrm{Max}}_{x\in X}}\mathop{\mathrm{Min}}_{y\in Y}\Phi(x,y) = f_{\leq} \stackrel{\text{def}}{=} \mathrm{Max}\bigcup_{x\in X}\bigcap_{y\in Y}\{\psi\in \mathbb{R}^Q|\psi\leq\Phi(x,y)\}$$

respectively [2]. Here and elsewhere in righthand sides, Max (or Min) means the set of Slater maximal (or minimal) elements in criterial space.

Similarly for minimax and maximin of the MZSG Player II:

$$\overline{\mathop{\mathrm{Min}}_{y\in Y}}\mathop{\mathrm{Max}}_{x\in X}\Phi(x,y) = \mathcal{F}_{\geq} \stackrel{\text{def}}{=} \mathrm{Min}\bigcup_{y\in Y}\bigcap_{x\in X}\{\psi\in \mathbb{R}^Q|\psi\geq\Phi(x,y)\}$$

$$\text{and}\quad \mathop{\mathrm{Max}}_{x\in X}\overline{\mathop{\mathrm{Min}}_{y\in Y}}\Phi(x,y) = f_{\geq} \stackrel{\text{def}}{=} \mathrm{Min}\bigcap_{x\in X}\bigcup_{y\in Y}\{\psi\in \mathbb{R}^Q|\psi\geq\Phi(x,y)\}.$$

In MAG, her minimax is $\mathcal{F}_{\not<} \stackrel{\text{def}}{=} \mathrm{Min}\bigcup_{y\in Y}\bigcap_{x\in X}\{\psi\in \mathbb{R}^Q|\psi\not<\Phi(x,y)\}$ and her maximin is $f_{\not<} \stackrel{\text{def}}{=} \mathrm{Min}\bigcap_{x\in X}\bigcup_{y\in Y}\{\psi\in \mathbb{R}^Q|\psi\not<\Phi(x,y)\}$.

Theorems:

1 [2]: $f_{\leq} = f_{\not<}$ and $\mathcal{F}_{\leq} = \mathcal{F}_{\not<}$, i.e., a MAG of perfect information has a solution.

2. $f_{\leq}\bigcap f_{\geq}\neq\emptyset$ iff Pareto values of $f_{\leq}$ and $f_{\geq}$ are singletons; $\mathcal{F}_{\leq}\bigcap\mathcal{F}_{\geq}\neq\emptyset$ iff Pareto values of $\mathcal{F}_{\leq}$ and $\mathcal{F}_{\geq}$ are singletons; i.e., as a rool, a MZSG of perfect information has no solution.

3. For continuous $\Phi\geq 0$ and compact sets the equality $f_{\leq}=\mathcal{F}_{\leq}$ is valid iff $\forall\mu\in M \stackrel{\text{def}}{=} \{\mu\geq 0|\sum_{i=1}^{Q}\mu_i = 1\}$

$$\min_{y\in Y}\max_{x\in X}\min_{i\in I(\mu)}\varphi_i(x,y)/\mu_i = \max_{x\in X}\min_{y\in Y}\min_{i\in I(\mu)}\varphi_i(x,y)/\mu_i,$$

where $I(\mu)\stackrel{\text{def}}{=}\{i=\overline{1,Q}|\ \mu_i\neq 0\}$. This implicates that the equalities of maximin and minimax for all partial criteria are not enough for equality of vector maximin and minimax. (There are examples that the set M of parameters μ cannot be reduced a priori.)

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References

1. G. Jentzsch. Some thoughts on the theory of cooperative games // Advances in game theory. Ann. Math. Studies. 1964. V.52. P. 407–442.

2. I.I. Pospelova. A classification for the problems of vector optimization under uncertainty // Comp. Maths Math. Phys. 2000. V.40, N.6. P. 820–836.

Dynamic Multicriterial Antagonistic Games with Fixed Order of Moves

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Multicriterial antagonistic game (MAG) is a two-person game $\langle X, Y, \Phi\rangle$ with a vector-function $\Phi = (\varphi_1, \dots, \varphi_Q)$ of payoff where the 1st plaeyr wants to maximize (in the criterial space) the set $\{\psi \in \mathbb{R}^Q | \psi \le \Phi(x, y)\}$ by choosing $x \in X$, but the 2nd player wants to minimize (in the criterial space) the set $\{\xi \in \mathbb{R}^Q | \xi \not< \Phi(x, y)\}$ by choosing $y \in Y$ [1]. Here and elsewhere usual inequality signs between vectors are interpreted componentwise, a crossed inequality means negation of the vector inequality, and Max (Min) are considered with respect to $>$ $(<)$ over vectors in $\mathbb{R}^Q$. In the MAG the 2nd player uses weak estimates, i.e., she is interested in minimizing an arbitrary component of the payoff vector, while the 1st player is interested in maximizing all the components of Φ. Thus they are antagonists.

Now, let us consider a dynamic MAG (DMAG) with discrete time $t = 1, 2, \dots, T$. In the DMAG, the players' strategies are $x = (x^1, \dots, x^T)$ and $y = (y^1, \dots, y^T)$ where $x^1 \in X^1$, $y^1 \in Y^1(x^1)$, $x^2 \in X^2(x^1, y^1), \dots, x^{t+1} \in X^{t+1}(x^1, \dots, x^t, y^1, \dots, y^t)$, $y^{t+1} \in Y^{t+1}(x^1, \dots, x^t, x^{t+1}, y^1, \dots, y^t), \dots$, $x^T \in X^T(x^1, \dots, x^{T-1}, y^1, \dots, y^{T-1})$, and $y^T \in Y^T(x, y^1, \dots, y^{T-1})$. Denote by $\mathbf{x}^t = (x^1, \dots, x^t)$ and $\mathbf{y}^t = (y^1, \dots, y^t)$ the parts of strategies chosen to the moment t. Assume that at the first step of DMAG the 1st payer moves by choosing $x^1 \in X^1$; after that the 2nd player chooses $y^1 \in Y^1(x^1)$. Knowing y^1, the 1st player makes her next move $x^2 \in X^2(x^1, y^1)$, and so on. At step $t + 1$ the 1st player knows $(\mathbf{x}^t, \mathbf{y}^t)$ and the 2nd player knows $(\mathbf{x}^{t+1}, \mathbf{y}^t)$, i.e., the players of the game have perfect information.

Basing on the results of [2], we define the best guaranteed estimation of the 1st player's payoff as follows: $W_1 =$

$$\mathrm{Max} \bigcup_{x^1\in X^1} \bigcap_{y^1\in Y^1(x^1)} \dots \bigcup_{x^T\in X^T(\mathbf{x}^{T-1},\mathbf{y}^{T-1})} \bigcap_{y^T\in Y^T(x,\mathbf{y}^{T-1})} \{\psi \leq \Phi(x,y)\}.$$

Similarly, the best weak estimation of the 2nd player's payoff in the DMAG is defined by $W_2 =$

$$\mathrm{Min} \bigcap_{x^1\in X^1} \bigcup_{y^1\in Y^1(x^1)} \dots \bigcap_{x^T\in X^T(\mathbf{x}^{T-1},\mathbf{y}^{T-1})} \bigcup_{y^T\in Y^T(x,\mathbf{y}^{T-1})} \{\xi \not< \Phi(x,y)\}.$$

By induction, using Th.4 from [3,4], we prove

Assertion: $W_1 = W_2$, i.e., the DMAG with the fixed order of moves has a solution.

The other variant of DMAG with fixed order of moves appears if the 2nd player makes the first move. Then the players' payoffs are estimated by

$$W_1' = \mathrm{Max} \bigcap_{y^1\in Y^1} \bigcup_{x^1\in X^1(y^1)} \bigcap_{y^2\in Y^2(x^1,y^1)} \dots \bigcup_{x^T\in X^T(\mathbf{x}^{T-1},y)} \{\psi \leq \Phi(x,y)\},$$

$$W_2' = \mathrm{Min} \bigcup_{y^1\in Y^1} \bigcap_{x^1\in X^1(y^1)} \bigcup_{y^2\in Y^2(x^1,y^1)} \dots \bigcap_{x^T\in X^T(\mathbf{x}^{T-1},y)} \{\xi \not< \Phi(x,y)\},$$

respectively. It is also valid that $W_1' = W_2'$.

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References

1. N.M. Novikova, I.I. Pospelova, and E.M. Kreines. Two Models for Multicriterial Games // This volume. (2001)

2. N.M. Novikova, I.I. Pospelova, and A.S. Semovskaya. Multistage Vector Minimax // Comp. Maths Math. Phys., 2000. V.40, N.10. P. 1391–1403.

3. N.M. Novikova, I.I. Pospelova. Multicriterial Problems of Decision Making under Uncertainty. Moscow: Computing Center of RAS. 2001. 64 p. [In Russian].

4. Pospelova, I.I. A classification for the problems of vector optimization under uncertainty, Comp. Maths Math. Phys., V.40, N.6 (2000).