Global minimization of rational functions using semidefinite programming

Etienne de Klerk[†], Dorina Jibetean[‡]

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Rational function minimization

Let $p, q, p_1, \ldots, p_k \in \mathbb{R}[x_1, \ldots, x_n]$ (polynomials with real coefficients defined on $\mathbb{R}^n$) with p and q relatively prime.

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$$p^* := \inf_{x \in S} \frac{p(x)}{q(x)}$$

where S is the *semi-algebraic set* given by

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p^* is not necessarily attained or finite!

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- H_2 model reduction (D. Jibetean. PhD Thesis, CWI, Amsterdam, 2003.);
- stability analysis of certain dynamical systems, including biochemical reactor models.

Possible approaches

- If the infimum is attained one can solve the first order optimality condition equations. Modern review: B. Sturmfels, *Solving Systems of Polynomial Equations*, AMS, 2002. If the inf is not attained ...

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- Global optimization codes — can converge to local minima.
- Today's talk: approaches involving semidefinite programming (SDP).

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If the data matrices diagonal $\Rightarrow$ LP

Different cases

We investigate SDP-based approaches for the following cases of $\inf_{x \in S} p(x)/q(x)$:

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- $S = \mathbb{R}^n$ and $n = 1$ (Unconstrained minimization: univariate case);
- $S = \mathbb{R}^n$ and general n (Unconstrained minimization: general case);
- S is compact, connected and general n (Constrained case);

Unconstrained case

Consider the unconstrained problem.

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p^* \; &:= \; \inf_{x \in \mathbb{R}^n} \frac{p(x)}{q(x)} \\
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We can replace the nonnegativity condition by a simpler one ...

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This leads us to the theory of *nonnegative polynomials*.

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- $n = 1$ (univariate polynomials) (result by Markov?);
- $d = 2$ (quadratic polynomials on n variables);
- $n = 2$ and $d \leq 4$ (bivariate polynomials of degree at most 4) (result by Hilbert);

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In all other cases counterexamples exist.

The sum of squares cone

We fix a basis of monomials

$$\tilde{x}_{n,d} := (1, x_1, \ldots, x_n, x_1^2, \ldots, x_n^d) \ \text{dim:} \binom{n+d}{d}.$$

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Notation: We denote the convex cone generated by squares of polynomials on $\mathbb{R}^n$ of degree at most d by $\Sigma^2_{n,2d}$ (*sum-of-squares (SOS) cone*).

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(We drop the subscripts when they are clear from the context.)

The sum of squares cone (cdt.)

Theorem: For a given $f \in \mathbb{R}[x_1, \ldots, x_n]$ of degree $2d$, one has $f \in \Sigma^2_{n,2d}$ iff

$$f = \tilde{x}_{n,d}^T M \tilde{x}_{n,d}$$

for some $M \succeq 0$ (size $\binom{n+d}{d} \times \binom{n+d}{d}$).

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cf. Theorem 17.1 in Y. Nesterov. Squared functional systems and optimization problems. In J.B.G. Frenk et al. eds., *High performance optimization*, 405–440. KAP, 2000.

Unconstrained univariate case

If q is nonnegative on $\mathbb{R}$, then

$$\begin{aligned} \inf_{x\in\mathbb{R}} \frac{p(x)}{q(x)} &= \sup_{t,x} \{t \;:\; p(x) - tq(x) \geq 0 \;\forall x \in \mathbb{R}\} \\ &= \sup_{t,x} \{t \;:\; p(x) - tq(x) \in \Sigma^2\} \\ &= \sup_{t,x} \{t \;:\; p(x) - tq(x) = \tilde{x}^T M \tilde{x}\} \end{aligned}$$

for some $M \succeq 0$, where

$$\tilde{x}^T = [1 \; x \; x^2 \ldots x^{\frac{1}{2}\max\{\deg(p),\deg(q)\}}].$$

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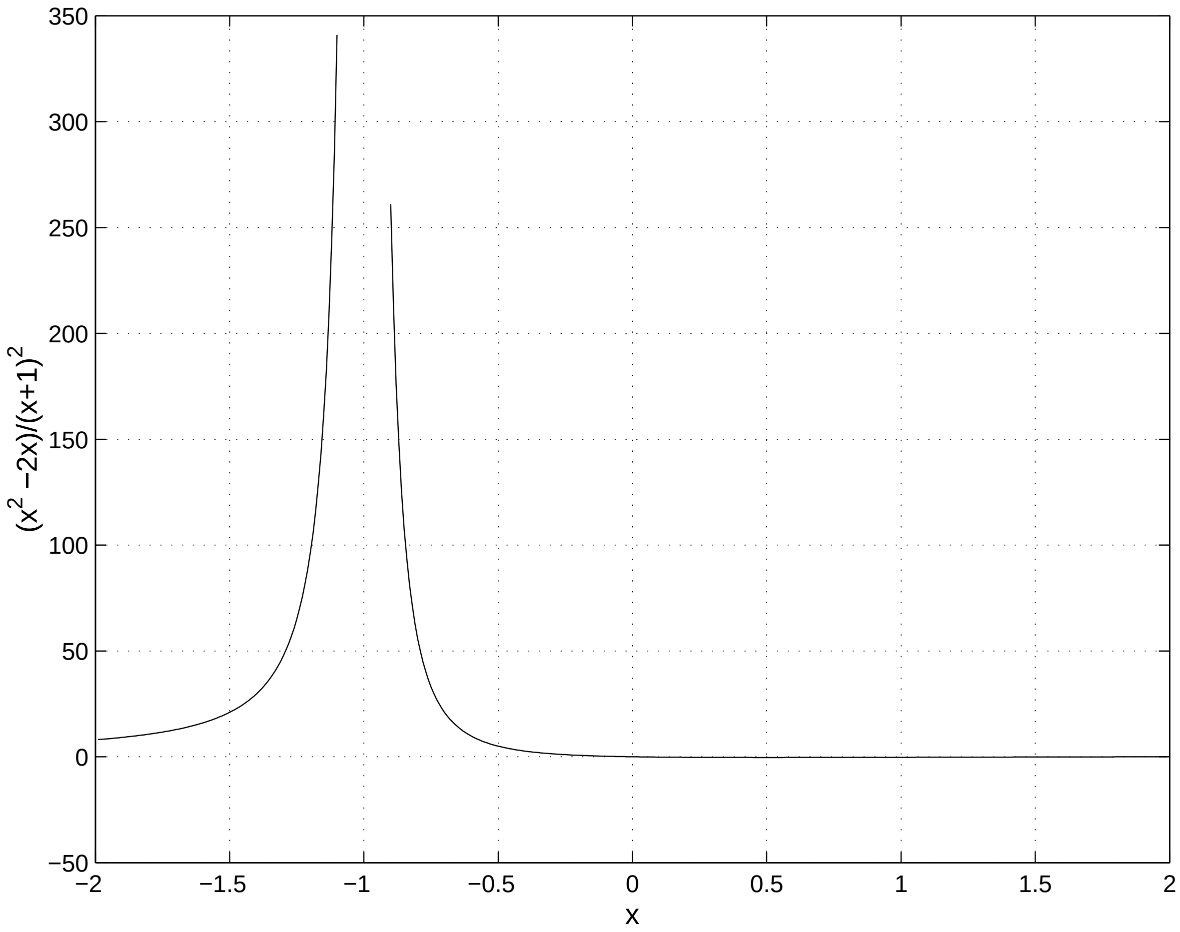
This is an SDP problem! (Result already obtained by Nesterov for $q(x) \equiv 1$.)

Y. Nesterov. Squared functional systems and optimization problems. In J.B.G. Frenk et al. eds., *High performance optimization*, 405–440. KAP, 2000.

Example

$$\frac{p(x)}{q(x)} := \frac{x^2 - 2x}{(x+1)^2}.$$



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Equivalent problem: $\sup t$ such that

$$(1-t)x^2 - 2(1+t)x - t = \begin{bmatrix} 1 \\ x \end{bmatrix}^T \begin{bmatrix} M_{00} & M_{01} \\ M_{10} & M_{11} \end{bmatrix} \begin{bmatrix} 1 \\ x \end{bmatrix}, \tag{2}$$

for some $M \succeq 0$.

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From (2):

$$M_{00} = -t, \quad M_{01} = M_{10} = -(1+t), \quad M_{11} = 1 - t.$$

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$$\min_{x \in \mathbb{R}} \frac{p(x)}{q(x)} = \max_{t,M} t$$

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Note that the optimal value is $p^* = -1/3$.

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Consider a *semi-algebraic set*

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One can treat the *unconstrained multivariate problem* by adding an artificial constraint $\|x\|^2 \leq R$ for some 'large' R.

Constrained case

Theorem (Jibetean) Assume that S is open and connected (or the (partial) closure of such a set). If $p^* > -\infty$ then q does not change sign on S. Assuming $q(x) \geq 0$ on S, then

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Consequence

$$\inf_{x \in S} \frac{p(x)}{q(x)} = \sup \{\rho \; : \; p(x) - \rho q(x) \geq 0 \;\; \forall x \in S\}.$$

Constrained multivariate case

Technical assumption: S is compact and there exists a

$$\bar{p} \in \Sigma^2 + p_1\Sigma^2 + \ldots + p_k\Sigma^2$$

such that $\{x \ : \ \bar{p}(x) \geq 0\}$ is *compact*.

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Theorem (Putinar): For a given polynomial p_0 one has $p_0(x) > 0$ for all $x \in S$ iff

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M. Putinar. Positive polynomials on compact semi-algebraic sets. *Ind. Univ. Math. J.* 42:969–984, 1993.

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If p and q have no common roots in S, then by Putinar's and Jibetean's theorems:

$$\begin{aligned} p^* &= \sup\{\rho \ : \ p(x) - \rho q(x) > 0 \ \ \forall x \in S\} \\ &= \sup\left\{\rho \ : \ (p - \rho q) \in \Sigma^2 + p_1\Sigma^2 + \ldots + p_k\Sigma^2\right\} \\ &\geq \sup\left\{\rho \ : \ (p - \rho q) \in \Sigma^2_{n,t} + p_1\Sigma^2_{n,t} + \ldots + p_k\Sigma^2_{n,t}\right\} \\ &:= \rho_t \ \ \text{(for any integer } t \geq 1\text{)}. \end{aligned}$$

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Computation of ρ_t: SDP problem with matrices of size $\binom{n+t}{t} \times \binom{n+t}{t}$ and at most $\max\{\deg(p), \deg(q)\}$ constraints — "polynomial" complexity for $t = O(1)$.

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These results by already obtained by Lasserre for $q(x) \equiv 1$ (polynomial objective function).

J.B. Lasserre. Global optimization with polynomials and the problem of moments. *SIOPT*, 11:296–817, 2001.

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Now we have $\min_{x \in S} \frac{p(x)}{q(x)}$ where S is the compact semi-algebraic set

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No a priori choice for R available in general.

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GloptiPoly and SOStools extremely useful to prove *global optimality* in small problems.

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- SDP approach competitive with state-of-the-art global optimization software.
- Need for large-scale (parallel?) SDP solvers to solve the large SDP relaxations.