Chaotic Dynamics on a Simplex and Global Optimization Methods with Normalized Constraints

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The application of chaos to global optimization methods are, 1) maps with respect to inner variables derived by discretizing gradient method models with Euler's method are unstabilized by setting their sampling time large, 2) chaotic trajectories of the optimizer's variables confined in the bounded searching domain are generated by nonlinear transformations of the unstabilized inner variables, and 3) the chaotic annealing method is available which conversely stabilize their dynamics by gradually decreasing the sampling time of them. However, the efficiency of the above-mentioned method called the chaotic global optimization was only reported for optimization problems constrained by upper and lower bounds,

$$\min_{\boldsymbol{x}} \quad E(\boldsymbol{x}), \tag{1a}$$

$$\text{subj.to} \quad p_i \leq x_i \leq q_i, \quad p_i, q_i \in R, \quad i = 1, \cdots, n. \tag{1b}$$

In this paper, to the contrary, we attempt to apply the method to optimization problems with normalized equality and non-negativity constraints,

$$\min_{\boldsymbol{x}} \quad E(\boldsymbol{x}), \tag{2a}$$

$$\text{subj.to} \quad \sum_{i=1}^{n} x_i = 1, \tag{2b}$$

$$x_i \geq 0, \quad i = 1, \cdots, n. \tag{2c}$$

First, based on the replicator model which is regarded as the gradient projection method with a variable metric, two types of chaotic maps on a simplex are presented. The one is a steepest gradient model with respect to inner state variables

$$\frac{du_i(t)}{dt} = -\frac{\partial E(\boldsymbol{x}(t))}{\partial x_i}, \tag{3a}$$

$$x_i(t) = \frac{\exp u_i(t)}{\sum_{j=1}^{n} \exp u_j(t)}, \tag{3b}$$

to which the replicator model

$$\frac{dx_i(t)}{dt} = -x_i(t) \left\{ \frac{\partial E(\boldsymbol{x}(t))}{\partial x_i} - \sum_{j=1}^{n} x_j(t) \frac{\partial E(\boldsymbol{x}(t))}{\partial x_j} \right\} \tag{4}$$

is equivalently transformed. The other is a replicator model for an unconstrained problem with respect to inner state variables obtained by variable transformation

$$\frac{dy_i(t)}{dt} = -x_i(t) \left\{ \frac{\partial E(\boldsymbol{x}(t))}{\partial x_i} - \sum_{j=1}^{n} x_j(t) \frac{\partial E(\boldsymbol{x}(t))}{\partial x_j} \right\}, \tag{5a}$$

$$x_i(t) = \frac{\exp y_i(t)}{\sum_{j=1}^{n} \exp y_j(t)}. \tag{5b}$$

Moreover, both discretized maps derived from the dynamical models (3) or (5) by applying Euler's differenciation method generate chaos if its sampling time is set large and equilibrium points of the dynamical systems are unstabilized. Chaotic trajectories of them are efficient for global search without being trapped onto local minima.

On the other hand, in order to find the global optimum, the chaotic annealing method, which decreases the sampling time of the discretized maps from the chaotic phase to the steepest descent one to stabilize the equilibrium points of the system and converge their trajectories to the global optimum. Especially, the hybrid type of the chaotic annealing method, which combines the accepting method to eliminate the redundant state transition that vastly increases the objective function's value to the normal chaotic annealing method, has the outstanding ability to find the global optimum with high rates.

In the paper, we demonstrate the global bifurcation structure of the discretized gradient maps on simplices and show the efficiency of the hybrid type of the chaotic annealing method with numerical simulations for a few problems.