## Numerical Optimization

**Objective function** $f(\boldsymbol{x})$ (formula or simulation algorithm)
- scalar function to be optimized
- multi-objectives can be combined into a single function

**Variables (unknowns)** $x$
- reals ⇒ **continuous** optimization
- discrete ⇒ **combinatorial** optimization

**Constraints** $g_i(\boldsymbol{x}) = 0,\ h_i(\boldsymbol{x}) \geq 0$
- equality, inequality, no constraints
- can be embedded into objective function

A.k.a. **nonlinear programming** problem

## The Optimization Problem

Find values of the variables that minimize (or maximize) the objective function while satisfying the constraints.

- To maximize, can minimize $-f(\boldsymbol{x})$ or $1/f(\boldsymbol{x})$

- Problem is often difficult or intractable

⇒ many specific methods have been devised for particular kinds of problems (e.g., linear, quadratic, integer, and mixed-integer programming, local optimization)

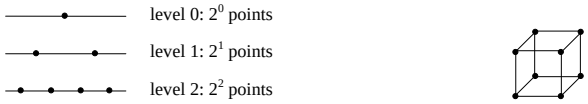
But there are also heuristic methods for general problems of **global optimization**

## Some Global Optimization Methods

- branch and bound
- continuation/smoothing
- dynamic programming
- evolutionary algorithms
- exhaustive search
- hybrid global/local
- multistart
- random search
- simplex
- simulated annealing
- tabu search

## Curse of Dimensionality

Exhaustive search using multidyadic levels of resolution




For $n$ dimensions need $(2^n)^l$ function evaluations at level $l$

For $l = 0$ to $l_{max}$
  For all $\boldsymbol{x} \in$ level $l$
    If $f(\boldsymbol{x}) < f(\boldsymbol{x}_{best\_so\_far})$
      $\boldsymbol{x}_{best\_so\_far} \leftarrow \boldsymbol{x}$

## Curse of Dimensionality

Suppose we can do $10^6$ function evaluations per second.
How long does it take to exhaustively search at level 1 these numbers of dimensions:

| $n$ | Time (seconds, minutes, hours, days, weeks, etc.) |
|---|---|
| 20 | |
| 26 | |
| 32 | |
| 36 | |
| 39 | |
| 45 | |
| 51 | |
| 74 | |

Frame of reference:
Earth life ≈ $3.5(10)^9$ years

## Supercomputers

- Workstations becoming faster and cheaper
- Supercomputers still wanted for modeling weather, defense, etc.
- Trend is to cluster numerous nodes

| Players | Year Available | Speed |
|---|---|---|
| Silicon Graphics/Cray, NASA | present | 100 Gflops |
| SKY Computers, DOD | < 2003 | 256 Gflops |
| IBM, Lawrence Livermore | present | 3.88 Tflops |
| IBM, Intel, SG/Cray, Sun, | 2001 | 30 Tflops |
| Los Alamos, Sandia, LL | 2004 | 100 Tflops |