Some Comments on Morel and Renvoise's 'Global Optimization by Suppression of Partial Redundancies'

Arthur Sorkin
Tesuji, Inc.
12340 Indian Trail Road
Los Gatos, CA 95030-9403
art@fuseki.com

Abstract: There has been much continuing interest in the global optimization method of Morel and Renvoise as extended by Joshi-Dhamdere and Chow. Unfortunately, the original formulation of Morel and Renvoise and those of Joshi-Dhamdere and Chow do not agree as to the form of the algorithm. The precise form of the algorithm's boolean equations is critical to the algorithm's performance and efficient implementation. Some of the problems with the algorithm noted by other authors can be solved by using a slightly different formulation of the algorithm.

Categories and Subject Descriptors: D.3.4 {Programming Languages} Processors - compilers; optimization

General Terms: Languages, Theory

Addittional Keywords and Phrases: code hoisting, partial redundancy suppression, Morel and Renvoise's algorithm

I. Morel and Renvoise's Algorithm

There has been much continuing interest in the global optimization method of Morel and Renvoise [1] as extended by Joshi-Dhamdere [2] and Chow [3]. For example, papers by Drechler and Stadel [4], Sorkin [5], Dhamdere [6], and Knoop, Rüthing and Steffan [7] deal with perceived problems with the original algorithm. Unfortunately, the original formulation of Morel and Renvoise and those of Joshi-Dhamdere [2] and Chow [3] do not agree as to the form of the algorithm. The precise form of the algorithm's boolean equations is critical to the algorithm's performance and efficient implementation. Some of the problems with the algorithm noted by other authors can be solved by using a slightly different formulation of the algorithm.

Morel and Renvoise presented the following system of Boolean equations:

AVAILABILITY

$$\left\{\begin{aligned} \mathrm{AVIN}_b &= \prod_{p \in \mathrm{Pred}(b)} \mathrm{AVOUT}_p \\ \mathrm{AVOUT}_b &= \mathrm{COMP}_b + \mathrm{AVIN}_b * \mathrm{TRANSP}_b \end{aligned}\right\}$$

ANTICIPABILITY

$$\begin{cases} \mathrm{ANTOUT}_b = \prod_{s \in \mathrm{Succ}(b)} \mathrm{ANTIN}_s \\ \mathrm{ANTIN}_b = \mathrm{ANTLOC}_b + \mathrm{ANTOUT}_b * \mathrm{TRANSP}_b \end{cases}$$

PARTIAL AVAILABILITY

$$\begin{cases} \mathrm{PAVIN}_b = \sum_{p \in \mathrm{Pred}(b)} \mathrm{PAVOUT}_p \\ \mathrm{PAVOUT}_b = \mathrm{COMP}_b + \mathrm{PAVIN}_b * \mathrm{TRANSP}_b \end{cases}$$

POTENTIAL PLACEMENT

$$\begin{cases} \mathrm{PPIN}_b = \mathrm{CONST}_b * \left(\mathrm{ANTLOC}_b + \mathrm{PPOUT}_b * \mathrm{TRANSP}_b\right) * \prod_{p \in \mathrm{Pred}(b)} \left(\mathrm{PPOUT}_p + \mathrm{AVOUT}_p\right) \\ \mathrm{CONST}_b = \mathrm{ANTIN}_b * \left(\mathrm{PAVIN}_b + \mathrm{TRANSP}_b * {\sim}\mathrm{ANTLOC}_b\right) \\ \mathrm{PPOUT}_b = \prod_{s \in \mathrm{Succ}(b)} \mathrm{PPIN}_s \end{cases}$$

INSERTION

$$\mathrm{INSERT}_b = \mathrm{XPPOUT}_b * \left({\sim}\left(\mathrm{AVOUT}_b + \mathrm{XPPIN}_b * \mathrm{TRANSP}_b\right)\right)$$

DELETION

$$\mathrm{DELETE}_b = \mathrm{ANTLOC}_b * \mathrm{XPPIN}_b$$

II. Joshi and Dhamdere’s Equations

Joshi and Dhamdere noted that the PPIN/PPOUT computation duplicated the ANTIN/ANTOUT computation, making the *ANTICIPABILITY* system of equations unnecessary. Their formulation of *POTENTIAL PLACEMENT* also eliminates the CONST factor, so the *PARTIAL AVAILABILITY* system of equations is also unused.

POTENTIAL PLACEMENT (Joshi-Dhamdere)

$$\begin{cases} \mathrm{PPIN}_b = \left(\mathrm{ANTLOC}_b + \mathrm{PPOUT}_b * \mathrm{TRANSP}_b\right) * \prod_{p \in \mathrm{Pred}(b)} \left(\mathrm{PPOUT}_p + \mathrm{AVOUT}_p\right) \\ \mathrm{PPOUT}_b = \prod_{s \in \mathrm{Succ}(b)} \mathrm{PPIN}_s \end{cases}$$

III. Chow's Equations

In Chow's formulation, the *ANTICIPABILITY* system of equations is present, but the CONST factor is simplified in the *POTENTIAL PLACEMENT* system.

POTENTIAL PLACEMENT (Chow)

$$\begin{cases} PPIN_b = CONST_b * \left(ANTLOC_b + PPOUT_b * TRANSP_b\right) * \prod_{p \in Pred(b)} \left(PPOUT_p + AVOUT_p\right) \\ CONST_b = ANTIN_b * PAVIN_b \\ PPOUT_b = \prod_{s \in Succ(b)} PPIN_s \end{cases}$$

In Chow's analysis of the algorithm, it was noted that the PAVIN factor in CONST is critical to limiting the distance that partial redundancies are hoisted. This is necessary in order to limit the lifetimes of hoisted expressions, which minimizes register contentition. The lack of this factor in the Joshi and Dhamdere formulation actually maximizes the distance that hoisted expressions are moved, which greatly increases register contention.

IV. Combined Equations

Joshi and Dhamdere were correct in eliminating the *ANTICIPABILITY* system of equations, and Chow was correct in retaining the PAVIN factor, so if we combine all three formulations of the algorithm, we get the following system of equations:

AVAILABILITY

$$\begin{cases} AVIN_b = \prod_{p \in Pred(b)} AVOUT_p \\ AVOUT_b = COMP_b + AVIN_b * TRANSP_b \end{cases}$$

PARTIAL AVAILABILITY

$$\begin{cases} PAVIN_b = \sum_{p \in Pred(b)} PAVOUT_p \\ PAVOUT_b = COMP_b + PAVIN_b * TRANSP_b \end{cases}$$

POTENTIAL PLACEMENT

$$\begin{cases} PPIN_b = PAVIN_b * \left(ANTLOC_b + PPOUT_b * TRANSP_b\right) * \prod_{p \in Pred(b)} \left(PPOUT_p + AVOUT_p\right) \\ PPOUT_b = \prod_{s \in Succ(b)} PPIN_s \end{cases}$$

This formulation of the algorithm has a number of advantages. First, it limits hoisting distance and, therefore, expression lifetimes and register contention. This is critical on real hardware where the number of registers available is limited. It also eliminates the unnecessary *ANTICIPABILITY* system of equations, so there is one less system of equations to compute. An analysis of the lifetimes of the terms of the above equations shows that only 7 booleans are in use per basic block at any one time, so the amount of space required to compute the equations is to 7 * BB * EXP bits, where BB is the number of basic blocks and EXP is the number of expressions.

The slow convergence of the bi-directional *POTENTIAL PLACEMENT* system of equations has been one of the major problems with the algorithm addressed by other authors. Perhaps the most interesting advantage of this formulation is that the PPIN equation is dominated by the PAVIN factor. This means that the *POTENTIAL PLACEMENT* system of equations can be treated as if it were a uni-directional system instead of a bi-directional system and, at least for "normal" flow graphs [5], will converge rapidly when iterated in bottom-up, depth-first order. It is not necessary, and, is, in fact, incorrect and inefficient to iterate bi-directionally.

The intuitive reason for this result is that for each expression, the PAVIN factor initializes to true precisely those bits for basic blocks where the expression *might* be moved. In the Joshi-Dhamdere formulation, for example, this information has to propagate from the basic blocks where ANTLOC is true via iteration of the booleans.

This formulation of the Morel and Renvoise algorithm can also be used to do strength-reduction by extending the definitions of COMP, ANTLOC and TRANSP in accordance with Chow [2]. It can also be extended to implement induction expression elimination.

REFERENCES

1. Morel, E. and Renvoise, C. Global Optimizattion by the Suppression of Partial Redundancies. *Commun. ACM* 22,2 (1979), 96-103.

2. Joshi, S. and Dhamdere, D. A Composite Hoisting-Strength Reduction Transformation for Global Program Optimization. *Int. J. Comp. Math* 11,1-2 (1982) 22-41,111-126.

3. Chow, F. A Portable Machine Independent Optimizer - Design and Meansurements. Ph.D. Dissertation, 1983, Department of Electrical Engineering and Technical Report 83-254, Computer Systems Laboratory, Stanford University.

4. Drechler, K. and Stadel, M. A Solution to a Problem with Morel and Renvoise's 'Global Optimization by Suppression of Partial Redundancies', *TOPLAS* 10,4 (1988) 635-640.

5. Sorkin, A. Some Comments on "A Solution to a Problem with Morel and Renvoise's 'Global Optimization by Suppression of Partial Redundancies", *TOPLAS* 11,4 (1989) 666-668.

6. Dhamdere, D. Practical Adaptation of the Global Optimization Algorithm of Morel and Renvoise, *TOPLAS* 13,2 (1991) 291-294.

7. Knoop, J., Rüthing, O., and Steffan, B. Optimal Code Motion: Theory and Practice, Universitat Passau, MIP-9310, December 1993 (to appear in *TOPLAS*).