Bohmian mechanics contradicts quantum mechanics

Arnold Neumaier

Institut für Mathematik, Universität Wien
Strudlhofgasse 4, A-1090 Wien, Austria
email: neum@cma.univie.ac.at
WWW: http://solon.cma.univie.ac.at/~neum/

Abstract.

It is shown that, for a harmonic oscillator in the ground state, Bohmian mechanics and quantum mechanics predict values of opposite sign for certain time correlations.

The discrepancy can be explained by the fact that Bohmian mechanics has no natural way to accomodate the Heisenberg picture, since the local expectation values that define the beables of the theory depend on the Heisenberg time being used to define the operators.

Relations to measurement are discussed, too, and shown to leave no loophole for claiming that Bohmian mechanics reproduces *all* predictions of quantum mechanics *exactly*.

Keywords: Bohmian mechanics, quantum mechanics, pilot wave, causal interpretation, local expectation value, harmonic oscillator, time correlations

1998 PACS Classification: 03.65.Bz
E-print Archive No.: quant-ph/0001011

1 Introduction

Since its inception by Bohm [6] and its popularization by Bell [3], the pilot wave theory, or causal interpretation of quantum mechanics – now often called Bohmian mechanics – has been regarded by a number of people as a in some respects bizarre but otherwise viable ontology for quantum mechanics. Books and proceedings appeared that discuss the features of the theory in detail, cf. Holland [20], Bohm & Hiley [7], Cushing et al. [9], good introductory surveys are available, cf. Berndl et al. [4], Dürr et al. [14], and accounts for the lay reader exist, cf. Albert [1], Goldstein [18].

On the other hand, Bohmian mechanics has remained a minority view, since, from its beginnings, it had been critically viewed by most of the influential quantum physicists. The main early arguments against it are stated in Holland [20, Sections 1.5.3 and 6.5.3]; they are usually argued away by some mathematical analysis accompanied by statements such as "classical prejudice" (Bell [3, Chapter 14]),"to our knowledge no serious technical objections have ever been raised against" it (Holland [20, Section 1.5.3]), or "Bohmian mechanics accounts for all of the phenomena governed by non-relativistic quantum mechanics" (Dürr et al. [14]). The arguments on both sides usually rest on one's unwillingness or readiness to accept counterintuitive consequences of the Bohmian picture, since none of the phenomena in question are observable.

More recent counterintuitive implications of Bohmian mechanics (Englert et al. [15], Griffiths [19]) met with similar responses (Dürr et al. [13], Dewdney et al. [11]). In particular, Dürr et al. write, "an open-minded advocate of quantum orthodoxy would presumably have preferred the clearer and stronger claim that BM is *incompatible* with the predictions of quantum theory, so that, despite its virtues, it would not in fact provide an explanation of quantum phenomena. The authors are, however, aware that such a strong claim would be false."

The purpose of this paper is to demonstrate – independent of the arguments in [15, 19, 20] – that such a strong claim is valid indeed. Specifically, Bohmian mechanics contradicts the predictions of quantum mechanics at the level of time correlations. Since time correlations can be observed experimentally via linear response theory (see, e.g., Reichl [27, Chapter 15.H]), Bohmian mechanics and quantum mechanics cannot be both valid.

Concerning discrepancies between Bohmian mechanics and quantum mechanics involving multiple times, see also Redington et al. [26] for Bohmian hydrogen atoms, and Ghose [17] for histories of indistinguishable particles.

There are similar problems with multiple times in NELSON's [25] stochastic quantum mechanics; however, there they can be overcome by a specific procedure for state reduction under measurement, see BLANCHARD et al. [5]. Bohmian mechanics does not seem to have such an option to rescue their case since in the orthodox Bohmian interpretation state reduction is a purely dynamical phenomenon.

Acknowledgments. I'd like to thank Philippe Blanchard, Sheldon Goldstein, Arkadiusz Jadzcyk and Jack Sarfatti for their comments on an earlier version of this paper.

2 Background

Quantum mechanics. A one-dimensional quantum particle without spin in an external potential $V(q)$ is described by the Hamiltonian

$$H(p,q) = \frac{p^2}{2m} + V(q) \tag{1}$$

(see, e.g., MESSIAH [24, (2.20)]), where the position operator q and the momentum operator p satisfy the canonical commutation relations

$$[q,p] = i\hbar \tag{2}$$

[24, (5.53)]. In the Schrödinger picture, observables are associated with Hermitian operators A. The dynamics of a quantity A is given in the Heisenberg picture by one-parameter families of operators $A(t)$ satisfying

$$i\hbar\dot{A}(t) = [A(t), H(p(t), q(t))] \tag{3}$$

[24, (8.40)]; the identification with the Schrödinger picture is obtained by specifying the initial condition $A(0) = A$ at some reference time $t = 0$.

In the position representation, pure ensemble states are given by wave functions $\psi_0(x)$ satisfying $\int |\psi_0(x)|^2 dx = 1$, on which q acts as multiplication by x and p acts as the differential operator $\frac{\hbar}{i}\nabla$. The expectation of a Heisenberg operator family $A(t)$ in a pure ensemble is defined by

$$\langle A(t)\rangle_Q = \int \psi_0^*(x)(A(t)\psi_0)(x)dx \tag{4}$$

[24, (4.22)]. If one defines a time-dependent wave function $\psi(x,t)$ as the solution of the initial-value problem

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = H\psi(x,t), \quad \psi(x,0) = \psi_0(x) \tag{5}$$

[24, (2.29)], one can rewrite the expectation in the equivalent Schrödinger picture as

$$\langle A(t)\rangle_Q = \int \psi^*(x,t)(A\psi)(x,t)dx \tag{6}$$

[24, (4.22)]. In particular, the expectation of a function of position is

$$\langle f(q(t))\rangle_Q = \int f(x)|\psi(x,t)|^2 dx \tag{7}$$

[24, (4.13)], so that

$$P(x,t) = |\psi(x,t)|^2 \tag{8}$$

[24, (4.2)] behaves as a probability density. For Hamiltonians of the form (1), the probability density satisfies an equation of continuity,

$$\frac{\partial}{\partial t}P + \operatorname{div} J = 0 \tag{9}$$

[24, (4.11)], with the probability current

$$J(x,t) = \operatorname{Re}\psi^*(x,t)\frac{\hbar}{im}\nabla\psi(x,t) \tag{10}$$

[24, (4.9)]. Thus an ensemble behaves like a flow of noninteracting particles.

Bohmian mechanics. Bohmian mechanics tries to give reality to this picture of an ensemble as a flow of particles with classical-like properties.

Following HOLLAND [20, Section 3.1], ensembles are interpreted in Bohmian mechanics as classical ensembles of particles characterized by a solution $\psi(x,t)$ of Schrödinger's wave equation (5) and a trajectory $x(t)$ obtained by solving the initial value problem

$$\dot{x}(t) = \frac{1}{m}\nabla S(x,t)\Big|_{x=x(t)}, \tag{11}$$

where the phase $S(x,t)$ of ψ is defined by

$$\psi(x,t) = e^{iS(x,t)/\hbar}|\psi(x,t)|. \tag{12}$$

The probability that a particle in the ensemble lies between the points x and $x+dx$ at time t is given by $|\psi(x,t)|^2dx$. (Holland discusses the 3-dimensional case and hence has a volume element in place of dx. It would be trivial to rewrite the present discussion in three dimensions without changing the conclusion. Similarly, as in many expositions of Bohmian mechanics, spin is ignored, but incorporating it would not change anything essential.)

To indicate the flow of individual particles in an ensemble described by a fixed solution $\psi(x,t)$ of the Schrödinger equation, we refine the notation and write $x_\xi(t)$ for the position of a particle that is in position ξ at time $t=0$, so that $x_\xi(0)=\xi$. The associated probability measure is then $d\mu(\xi)=|\psi_0(\xi)|^2d\xi$. Ensemble expectations of some real property A_ξ that a particle – characterized by its wave function ψ_0 (asumed fixed) and its position ξ at time $t=0$ – has are therefore given by averaging the values of A over the ensemble,

$$\langle A\rangle_B = \int A_\xi|\psi_0(\xi)|^2d\xi. \tag{13}$$

Since

$$J(x(t),t) = P(x(t),t)\dot{x}(t) \tag{14}$$

[20, (3.2.29)], the continuity equation (10) implies that expectations of functions $A(x(t),t)$ are invariant under a shift of the reference time $t=0$. (Note that other authors use the equation

$$\dot{x}(t) = J(x(t),t)/P(x(t),t) \tag{15}$$

in place of (11) to define the trajectories; because of (14), this is indeed equivalent and has the advantage of being directly motivated by time shift invariance.)

Local expectation values. To calculate expectation values of quantum mechanical operators, Holland [20, (3.5.4)] defines the local expectation value of a Hermitian operator A in the Schrödinger picture as the real number

$$A(x,t) = \mathrm{Re}\,\frac{(A\psi)(x,t)}{\psi(x,t)}. \tag{16}$$

The local expectation values evaluated along a trajectory,

$$A_\xi(t) = A(x_\xi(t),t), \tag{17}$$

are considered to be the real properties of a particle. Indeed, Holland mentions in [20, Section 3.7.2] that the local expectation value "might, following the common parlance, be termed the 'hidden variable' associated with the corresponding physical variable". With this definition of real properties, Bohmian mechanics achieves agreement with simple quantum mechanical predictions since, as is easily checked,

$$\langle A\rangle_B = \langle A\rangle_Q \tag{18}$$

(Holland [20, (3.8.8/9)]). To appreciate what the local expectation values are in specific cases, Holland calculates explicitly the case of position, momentum, total energy, and total orbital angular momentum. In particular, the particle positions (local expectation values of $A = q$) and particle momenta (local expectation values of $A = p$) at arbitrary times t are

$$q_\xi(t) = x_\xi(t), \quad p_\xi(t) = \nabla S(x_\xi(t), t) \tag{19}$$

[20, (3.2.18)]. More generally, if $A = f(q)$ then $A(x,t) = f(x)$; thus functions of position at a fixed time behave classically. But for other operators, this is not the case; e.g., while $p_\xi(t) = m\dot{x}_\xi(t)$, the kinetic energy $K = p^2/2m$ satisfies

$$K_\xi(t) = \frac{m}{2}\dot{x}_\xi(t)^2 + Q(x_\xi(t), t)$$

with an additional 'quantum potential' $Q(x,t)$.

3 Time correlations in Bohmian mechanics

Particles in the ground state. For any Hamiltonian with a nondegenerate ground state ψ_0 (satisfying $H\psi_0 = E_0\psi_0$), this ground state can always be taken to be real. Indeed, since the complex conjugate ψ_0^* also satisfies $H\psi_0^* = E_0\psi_0^*$ and the ground state is nondegenerate, ψ_0^* must be a multiple of ψ_0, and scaling with the square root of the multiplier leaves a real eigenfunction.

The solution ψ of the Schrödinger equation (5) corresponding to the ground state is

$$\psi(x,t) = e^{-itE_0/\hbar}\psi_0(x).$$

If a particle can be in position x at time t then $|\psi(x,t)|^2 > 0$, hence $\psi_0 \neq 0$. A comparison with (12) therefore shows that particles in a nondegenerate ground state have a phase $S(x,t) = \pm tE_0$ independent of x. Thus (11) implies that $x(t)$ is constant, $x_\xi(t) = \xi$ for all t. Thus each particle in the ensemble stands still.

This observation is puzzling and lead Einstein to reject the Bohmian interpretation; see HOLLAND [20, Section 6.5.3] for a discussion and a defense.

The harmonic oscillator. A one-dimensional harmonic oscillator of mass m, period T and angular frequency $\omega = 2\pi/T$ is described by the Hamiltonian

$$H(p,q) = \frac{p^2}{2m} + \frac{\omega^2 m}{2} q^2. \tag{20}$$

The canonical commutation relations (2) imply that, for the Hamiltonian (20), the Heisenberg dynamics (3) of position and momentum are given by

$$\frac{dq(t)}{dt} = \frac{p(t)}{m}, \quad \frac{dp(t)}{dt} = -\omega^2 m q(t),$$

just as in the classical case. In particular, we can solve the dynamics explicitly in terms of the position operator q and the momentum operator p at time $t = 0$ as

$$q(t) = q \cos \omega t + \frac{p}{\omega m} \sin \omega t,$$
$$p(t) = p \cos \omega t - q\omega m \sin \omega t,$$

again as in the classical case. In particular, $q(t + T/2) = -q(t)$, so that quantum mechanics predicts the time correlation

$$\langle q(t + T/2) q(t) \rangle_Q = -\langle q(t)^2 \rangle_Q < 0 \tag{21}$$

for an ensemble in an arbitrary pure (or even mixed) state. ($\langle q(t)^2 \rangle_Q = 0$ would be possible only in an eigenstate of $q(t)$ to the eigenvalue zero, but there is no such normalized state.)

On the other hand, interpreting the time correlations in a Bohmian sense, one finds from (19) and (13) that

$$\langle q(t + T/2) q(t) \rangle_B = \int q_\xi(t + T/2) q_\xi(t) |\psi_0(\xi)|^2 d\xi.$$

For particles in the ground state (which for the harmonic oscillator is nondegenerate), the discussion above shows that the right hand side is constant,

$$\langle q(t + T/2) q(t) \rangle_B = \langle q(t)^2 \rangle_B = \langle q(t)^2 \rangle_Q > 0. \tag{22}$$

Comparing (21) and (22), we see that the quantum mechanical time correlation and the Bohmian time correlation have opposite signs.

Measuring time correlations. The fact that, in general, $q(s)q(t)$ is not Hermitian and hence cannot be measured in *individual* events does not mean that the expectation on the left hand side of (21) is meaningless and has no relation to experiment. Indeed, one may define the expectation of an arbitrary quantity f in orthodox quantum mechanics (where all self-adjoint operators = observables can be measured, cf. DIRAC [12, p.37]) in terms of the observables $\mathrm{Re}\, f = \frac{1}{2}(f + f^*)$ and $\mathrm{Im}\, f = \frac{1}{2i}(f - f^*)$ by

$$\langle f \rangle_Q := \langle \mathrm{Re}\, f \rangle_Q + i \langle \mathrm{Im}\, f \rangle_Q. \tag{23}$$

This gives unambiguous values to all expectations, and is fully consistent with orthodox quantum mechanics. Of course, it may not be easy to measure $\mathrm{Re}\, f$ and $\mathrm{Im}\, f$, but an operational procedure for measuring arbitrary Hermitian functions of p and q by a suitable experimental arrangement can be found, e.g., in LAMB [22]. And quantum optics routinely deals with expectations and measurements of coherent states, which are eigenstates of nonhermitian annihilator operators; see, e.g. LEONHARDT [23].

While the example of the harmonic oscillator is somewhat artificial, it has the advantage that all calculations can be done explicitly. Significant physical applications of time correlations are, however, made in statistical mechanics, where integrals over time correlations in thermodynamic equilibrium states are naturally linked to linear response functions, and hence are measurable as susceptibilities. See, e.g., REICHL [27, (15.161) and (15.172)]. Time correlations also arise in the calculation of optical spectra (CARMICHAEL [8, Lecture 3.3]) and in the context of quantum Markov processes (GARDINER [16, Section 10.5]). Thus, at least in principle, it is possible to test the validity of the recipe (23) by experiment, by measuring susceptibilities or spectra directly, and by comparing the result to that obtained by applying (23) to measurements of $\mathrm{Re}\, f$ and $\mathrm{Im}\, f$.

As Arkadiusz Jadczyk (personal communication) pointed out, (23) implies that due to noncommutativity, the quantum mechanical time correlations $\langle q(s)q(t) \rangle_Q$ are complex in most states at most times, while time correlations computed from Bohm trajectories are always real. Thus an agreement would be a coincidence.

On the other hand, it is possible to avoid nonhermitian operators completely. Indeed, the contradiction persists in the following consequence of (21) and (22):

$$\langle q(t + T/2)q(t) + q(t)q(t + T/2) \rangle_Q = -2 \langle q(t)^2 \rangle_Q < 0, \tag{24}$$

$$\langle q(t + T/2)q(t) + q(t)q(t + T/2) \rangle_B = 2 \langle q(t)^2 \rangle_Q > 0. \tag{25}$$

Note that $q(t + T/2)q(t) + q(t)q(t + T/2)$ is Hermitian, and (24) has the correct classical time correlation as limit when $\hbar \to 0$. Symmetrized time correlations are discussed in the context of linear response theory in KUBO et al. [21, pp. 167-169].

In discussions with proponents of Bohmian mechanics, it is claimed that my interpretation of the Bohmian formalism is erroneous, in that I am not making the proper distinction between the ontological "beable" and the epistemological "observable", and compare the statistics of unobserved Bohm trajectories with those for quantum observations.

However, quantum mechanics can be used in practice without reference to the (still ill-defined) measurement mechanism, while Bohmian mechanics resorts to the latter to justify any discrepancy. This should not be the case if the 'beables' were the real entities that Bohmian mechanics claims them to be. And indeed, the whole purpose of the local expectation values is to show the equivalence of expectations in Bohmian mechanics with those in quantum mechanics, without having to refer to measurement.

What else could the meaning of (18) be? The whole discussion in HOLLAND [20, Section 3.5–3.8] becomes meaningless unless it is accepted that (18) is the real link between quantum mechanics and Bohmian mechanics, independent of any measurement questions. The probabilities – Holland discusses these independent of expectations – follow the rule (18) when A is an orthogonal projector corresponding to the associated subspaces, and if the expectation rule fails then associated probabilities also fail.

Thus, one wonders why Bohmian mechanics, which can do calculations of single-time probabilities without reference to measurement questions, suddenly needs the measurement process to calculate probabilities of pair events occuring at two different times.

It may be noted that there are similar problems with multiple times in NELSON's [25] stochastic quantum mechanics; BLANCHARD et al. [5] show how these problems can be overcome by a specific procedure for state reduction under measurement.

However, Bohmian mechanics does not seem to have such an option to rescue its interpretation since in the orthodox Bohmian interpretation, state reduction is a purely dynamical phenomenon. The suggestion to explain equivalence to quantummechanical predictions by invoking the measurement process leads at best to an approximate equivalence since Bohmian theory discusses measurement only in an approximate way (HOLLAND [20, Chapter 8], BOHM & HILEY [7, Chapter 6]). And even then, specific efforts would be needed to show that the time correlations come out in the right way.

And the explanation by measurement fails completely if we consider the universe as a whole which, if supposed to behave deterministically according to the laws of Bohmian mechanics, has no meaningful way of defining time correlations apart from $\langle q(s)q(t)\rangle_B$.

The ambiguity of local expectation values. To gain a better understanding of the problems of Bohmian mechanics from a slightly different point of view, we look more closely at the local expectation values that are supposed to define the real properties of particles, and that lie at the heart of the claim of Bohmian mechanics that all its predictions agree with those of quantum mechanics.

We first note that the recipe for calculating local expectation values is linear without restriction; in particular, for a particle in the ground state, where $q_\xi(t) = \xi$ and $p_\xi(t) = 0$, we have $A_\xi(t) = \alpha\xi$ for any operator $A = \alpha q + \beta p$. We use this to calculate the local expectation value of the Heisenberg position operator $A = q(s)$ at time s in the ground state of the harmonic oscillator, and find the remarkable formula

$$A_\xi(t) = \xi \cos \omega s.$$

Thus the objective value of $A = q(s)$ at any time t is $\xi \cos \omega s$, corresponding to our intuition if we regard s as the physical time. It seems that, at least in the Bohmian picture of the harmonic oscillator, the Heisenberg time s is the real time while the Schrödinger time t only plays a formal and counterintuitive role.

This gives weight to what is called 'operator realism' in Daumer et al. [10], against the Bohmian program advocated there. And it makes the interpretation of local expectation values as real properties of the system highly dubious since these values depend on the choice of the Heisenberg time s.

In particular, for multi-time expectations, which are meaningful only in the Heisenberg picture, there is no distinguished single Heisenberg time, and hence no natural Bohmian interpretation.

Thus Bohmian mechanics can at best be said to reproduce a subset of quantum mechanics. It contradicts the quantum mechanical predictions about time correlations if one proceeds in the straightforward way that generalizes the basic formula (18) that accounts for agreement of single time expectations and single-time probabilities.

And Bohmian mechanics does not say anything at all about time correlations if the connection to quantum mechanics is kept more vague and left hidden behind a measurement process that is inherently approximate in Bohmian mechanics. Should this be the real link between quantum mechanics and Bohmian mechanics, one could claim the predictions of Bohmian mechanics to be approximately equal onlyto those of quantum mechanics, against the explicit assertions of many supporters of Bohmian mechanics.

4 Conclusion

In contrast to the claim by Dürr et al. [14], Bohmian mechanics *does not* account for all of the phenomena governed by nonrelativistic quantum mechanics. Indeed, it was shown that for a harmonic oscillator in the ground state, Bohmian mechanics and quantum mechanics predict values of opposite sign for certain time correlations. Bohmian mechanics therefore contradicts quantum mechanics at the level of time correlations. Since time correlations can be observed experimentally via linear response theory, Bohmian mechanics and quantum mechanics cannot both describe experimental reality.

Due to the complicated form of the Bohmian dynamics, it seems difficult to compute time correlations for realistic scenarios where a comparison with linear response theory and hence with experiment would become possible. But perhaps numerical simulations are feasible. On the other hand, it is unlikely that, if the predictions of quantum mechanics and Bohmian mechanics differ in such a simple case, they would agree in more realistic situations.

The time correlations used in statistical mechanics are those from quantum mechanics and not those from Bohm trajectories. Moreover, they can be calculated and used without reference to any theory about the measurement process. If an elaborate theory of quantum observation is needed to reinterpret Bohmian mechanics – so that it matches quantum mechanics and thus restores the connection to statistical mechanics – then Bohmian mechanics is at best approximately equivalent to quantum mechanics and, I believe, irrelevant to practice.

It is therefore likely that Bohmian mechanics is ruled out as a possible foundation of physics.

References

[1] D.Z. Albert, Bohm's alternative to quantum mechanics. Scientific American 270 (May 1994), 32–39.

[2] L.E. Ballentine (ed.), Foundations of quantum mechanics since the Bell inequalities, Amer. Assoc. Physics Teachers, College Park, MD, 1988.

[3] J.S. Bell, Speakable and unspeakable in quantum mechanics, Cambridge Univ. Press, Cambridge 1987.

[4] K. Berndl, M. Daumer, D. Dürr, S. Goldstein and N. Zanghi. A Survey of Bohmian Mechanics, Il Nuovo Cimento 110B (1995), 737–750. quant-ph/9504010

[5] Ph. Blanchard, S. Golin and M. Serva, Repeated measurements in stochastic mechanics, Phys. Rev. D 34 (1986), 3732-3738.

[6] D. Bohm, A suggested interpretation of the quantum theory in terms of 'hidden' variables, I and II, Phys. Review 85 (1952), 166–179, 180–193. Reprinted in [28].

[7] D. Bohm and B.J. Hiley, The undivided universe, Routledge, London 1993.

[8] H. Carmichael, An open systems approach to quantum optics, Springer, Berlin 1993.

[9] J.T. Cushing, A. Fine and S. Goldstein, Bohmian mechanics and quantum theory: An appraisal, Kluwer, Dordrecht 1996.

[10] M. Daumer, D. Dürr, S. Goldstein and N. Zanghi, Naive Realism about Operators, Erkenntnis 45, 379–397 (1996). quant-ph/9601013

[11] C. Dewdney, L. Hardy and E.J. Squires, How late measurements of quantum trajectories can fool a detector, Physics Letters A 184 (1993), 6–11.

[12] P.A.M. Dirac, The principles of quantum mechanics, 4th ed., Clarendon Press, Oxford 1958.

[13] D. Dürr, W. Fusseder, S. Goldstein, and N. Zanghi, Comment on "Surrealistic Bohm trajectories", Z. Naturforsch. 48a (1993), 1261–1262.

[14] D. Dürr, S. Goldstein, and N. Zanghi, Bohmian mechanics as the foundation of quantum mechanics, pp. 21–45 in [9]. quant-ph/9511016

[15] B.-G. Englert, M.O. Scully, G. Süssmann and H. Walther, Surrealistic Bohm trajectories, Z. Naturforsch. 47a (1992), 1175–1186. Reply to Comment on "Surrealistic Bohm trajectories", Z. Naturforsch. 48a (1993), 1263–1264.

[16] C.W. Gardiner, Handbook of stochastic methods for physics, chemistry and the natural sciences, 2nd ed., Springer, Berlin 1985.

[17] P. Ghose, The incompatibility of the de Broglie-Bohm theory with quantum mechanics, Manuscript, 2000. quant-ph/0001024

[18] S. Goldstein, Quantum theory without observers, I and II, Physics Today, 1998, 42–46; April 1998, 38–42.

[19] R.B. Griffiths, Bohmian mechanics and consistent histories, Manuscript (1999). quant-ph/9902059

[20] P.R. Holland, The quantum theory of motion, Cambridge Univ. Press, Cambridge 1993.

[21] R. Kubo, M. Toda and N. Hashitsume, Statistical physics II. Nonequilibrium statistical mechanics, 2nd ed., Springer, Berlin 1991.

[22] W.E. Lamb, An operational interpretation of nonrelativistic quantum mechanics, Physics Today 22 (4) (1969), 23-28. Reprinted in [2].

[23] U. Leonhardt Measuring the quantum state of light, Cambridge Univ. Press, Cambridge 1997.

[24] A. Messiah, Quantum Mechanics, Vol. 1, North-Holland, Amsterdam 1991.

[25] E. Nelson, Quantum Fluctuations, Princeton Univ. Press, Princeton 1985.

[26] N. Redington, A. Widom and Y.N. Srivastava, Dipole moment noise in the hydrogen atom in the Bohm quantum-mechanical theory, Il Nuovo Cimento 109 (1994), 1169-1174.

[27] L.E. Reichl, A modern course in statistical physics, Edward Arnold, 1988.

[28] J.A. Wheeler and W. H. Zurek, Quantum theory and measurement. Princeton Univ. Press, Princeton 1983.