

Foundations of quantum physics

III. Measurement

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April 24, 2019

Abstract. This paper presents the measurement problem from the point of view of the thermal interpretation of quantum physics introduced in Part II. Unlike most work on the foundations of quantum mechanics, the present paper involves a multitude of connections to the actual practice of quantum theory and quantum measurement.

The measurement of a Hermitian quantity A is regarded as giving an uncertain value approximating the q -expectation $\langle A \rangle$ rather than (as tradition wanted to have it) as an exact revelation of an eigenvalue of A . Single observations of microscopic systems are (except under special circumstances) very uncertain measurements only.

The thermal interpretation

- treats detection events as a statistical measurement of particle beam intensity;
- claims that the particle concept is only asymptotically valid, under conditions where particles are essentially free.
- claims that the unmodeled environment influences the results enough to cause all randomness in quantum physics.
- allows one to derive Born's rule for scattering and in the limit of ideal measurements; but in general, only part of Born's rule holds exactly: Whenever a quantity A with zero uncertainty is measured exactly, its value is an eigenvalue of A ;
- has no explicit collapse – the latter emerges approximately in non-isolated subsystems;
- gives a valid interpretation of systems modeled by a quantum-classical dynamics;
- explains the peculiar features of the Copenhagen interpretation (lacking realism between measurements) and the minimal statistical interpretation (lacking realism for the single case) in the microscopic domain where these interpretations apply.

The thermal interpretation is an interpretation of quantum physics that is in principle refutable by theoretical arguments leading to a negative answer to a number of open issues collected at the end of the paper, since there is plenty of experimental evidence for each of the points mentioned there.

For the discussion of questions related to this paper, please use the discussion forum <https://www.physicsoverflow.org>.

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1 Introduction

This paper presents the measurement problem from the point of view of the **thermal interpretation** of quantum physics introduced in Part II [51] of this series.

In the thermal interpretation of quantum physics, the theoretical observables (the beables in the sense of BELL [6]) are the expectations and functions of them. They satisfy a deterministic dynamics. Some of these beables are practically (approximately) observable. In particular, q-expectations¹ of Hermitian quantities and q-probabilities, the probabilities associated with appropriate self-adjoint Hermitian quantities, are among the theoretical observables. The q-expectations are approximately measured by reproducible single measurements of macroscopic quantities, or by sample means in a large number of observations on independent, similarly prepared microscopic systems. The q-probabilities are approximately measured by determining the relative frequencies of corresponding events associated with a large number of independent, similarly prepared systems.

This eliminates all foundational problems that were introduced into quantum physics by basing the foundations on an unrealistic concept of observables and measurement. With the thermal interpretation, the measurement problem turns from a philosophical riddle into a scientific problem in the domain of quantum statistical mechanics, namely how the quantum dynamics correlates macroscopic readings from an instrument with properties of the state of a measured microscopic system.

This is the subject of the present paper. Everything physicists measure is measured in a thermal environment for which statistical thermodynamics is relevant. The thermal interpretation agrees with how one interprets measurements in thermodynamics, the macroscopic

¹ As in Part I [50] I follow the convention of ALLAHVERDYAN et al. [2], and add the prefix 'q-' to all traditional quantum notions that have in the thermal view a new interpretation and hence a new terminology. In particular, we use the terms q-observable, q-expectation, q-variance, q-standard deviation, q-probability, q-ensemble for the conventional terms observable, expectation, variance, standard deviation, probability, and ensemble.

part of quantum physics, derived via statistical mechanics. By its very construction, the thermal interpretation naturally matches the classical properties of our quantum world: The thermal interpretation assigns states – and a realistic interpretation for them – to individual quantum systems, in a way that large quantum systems are naturally described by classical observables.

Section 2 postulates a measurement principle that defines what it means in the thermal interpretation to measure a quantity with a specified uncertainty and discusses the role played by macroscopic systems and the weak law of large numbers in getting readings with small uncertainty. Since quantum physics makes many deterministic predictions, for example regarding observed spectra, but also assertions about probabilities, we distinguish deterministic and statistical measurements.

Unlike in traditional interpretations, single, nonreproducible observations do not count as measurements since this would violate the reproducibility of measurements – the essence of scientific practice. As a consequence, the measurement of a Hermitian quantity A is regarded as giving an uncertain value approximating the q -expectation $\langle A \rangle$ rather than (as tradition wanted to have it) as an exact revelation of an eigenvalue of A . This difference is most conspicuous in the interpretation of single discrete microscopic events. Except in very special circumstances, these are not reproducible. Thus they have no scientific value in themselves and do not constitute measurement results. Scientific value is, however, in ensembles of such observations, which result in approximate measurements of q -probabilities and q -expectations.

Since relativistic quantum field theory is the fundamental theory of elementary particles and fields, with the most detailed description, the simpler quantum mechanics of particles is necessarily a derived description. Section 3 discusses the extent to which a particle picture of matter and radiation is appropriate – namely in scattering processes, where particles may be considered to be essentially free except during a very short interaction time, and in the domain where the geometric optics perspective applies.

In 1852, at a time when Planck, Einstein, Bohr, Heisenberg, Schrödinger, Born, Dirac, and von Neumann – the founders of modern quantum mechanics – were not even born, George Stokes described all the modern quantum phenomena of a single qubit, explaining them in classical terms. Remarkably, this description of a qubit (recounted in Subsection 3.5) is fully consistent with the thermal interpretation of quantum physics. Stokes' description is couched in the language of optics – polarized light was the only quantum system that, at that time, was both accessible to experiment and quantitatively understood. Stokes' classical observables are the functions of the components of the coherence matrix, the optical analogue of the density operator of a qubit, just as the thermal interpretation asserts.

Section 4 gives the thermal interpretation of statistical mechanics. All statistical mechanics is based on the concept of coarse-graining, introduced in Subsection 4.2. Due to the neglect of high frequency details, coarse-graining leads to stochastic features, either in the models themselves, or in the relation between models and reality. Deterministic coarse-grained models are usually chaotic, introducing a second source of randomness. The most important form of coarse-graining leads to statistical thermodynamics of equilibrium and nonequilibrium, leading for example to the Navier–Stokes equations of fluid mechanics. Other ways

of coarse-graining lead to quantum-classical models, generalizing the Born–Oppenheimer approximation widely used in quantum chemistry.

A multitude of interpretations of quantum mechanics exist; most of them in several variants. As we shall see in Section 5, the mainstream interpretations may be regarded as partial versions of the thermal interpretation. In particular, certain puzzling features of both the Copenhagen interpretation and the statistical interpretation get their explanation through the thermal interpretation of quantum field theory. We shall see that these peculiar features get their natural justification in the realm for which they were created – the statistics of few particle scattering events.

The bulk of this paper is intended to be nontechnical and understandable for a wide audience being familiar with some traditional quantum mechanics. The knowledge of some basic terms from functional analysis is assumed; these are precisely defined in many mathematics books. However, a number of remarks are addressed to experts and then refer to technical aspects explained in the references given.

In the bibliography, the number(s) after each reference give the page number(s) where it is cited.

Acknowledgments. Earlier versions of this paper benefitted from discussions with Rahel Knöpfel.

2 The thermal interpretation of measurement

To clarify the meaning of the concept of measurement we postulate in Subsection 2.1 a measurement principle that defines what it means in the thermal interpretation to measure a quantity with a specified uncertainty.

The essence of scientific practice is the reproducibility of measurements, discussed in Subsection 2.2. The next two subsections distinguish deterministic and statistical measurements depending on whether a single observation is reproducible, and discuss the role played by macroscopic systems and the weak law of large numbers in getting readings with small uncertainty. The special case of event-based measurements described in terms of POVMs is considered in Subsection 2.5.

2.1 What is a measurement?

According to the thermal interpretation, properties of the system to be measured are encoded in the state of the system and its dynamics. This state and what can be deduced from it are the only objective properties of the system. On the other hand, a measuring instrument measures properties of a system of interest. The measured value – a pointer reading, a sound, a counter value, etc. – is read off from the instrument, and hence is

primarily a property of the measuring instrument and not one of the measured system. From the properties of the instrument (the instrument state), one can measure or compute the measurement results. Measurements are possible only if the microscopic laws imply quantitative relations between properties of the measured system (i.e., the system state) and the values read off from the measuring instrument (properties of the detector state).

This – typically somewhat uncertain – relation was specified in the rule (M) from Subsection 4.2 of Part I [50] that we found necessary for a good interpretation:

(M) We say that a property P of a system S (encoded in its state) has been **measured** by another system, the **detector** D , if at the time of completion of the measurement and a short time thereafter (long enough that the information can be read by an observer) the detector state carries enough information about the state of the measured system S at the time when the measurement process begins to deduce with sufficient reliability the validity of property P at that time.

To give a precise formal expression for rule (M) in the context of the thermal interpretation, we have to define the property P as the validity or invalidity of a specific mathematical statement $P(\rho_S)$ about the state ρ_S of the system and the information to be read as another specific mathematical statement $Q(\rho_D)$ about the state ρ_D of the detector. Then we have to check (theoretically or experimentally) that the dynamics of the joint system composed of system, detector, and the relevant part of the environment implies that, with high confidence and an appropriate accuracy,

$$Q(\rho_D(t)) \approx P(\rho_S(t_i)) \quad \text{for } t_f \leq t \leq t_f + \Delta t. \quad (1)$$

Here t_i and t_f denote the initial and final time of the duration of the measurement process, and Δt is the time needed to read the result.

For example, to have sufficient reasons to call the observation of a pointer position or a detector click an observation of a physical property of the measured system one must show that (1) holds for some encoding of the pointer position or detector click as $Q(\rho_B)$ and the property $P(\rho_S)$ claimed to be measured.

Establishing such a relation (1) based on experimental evidence requires knowing already how system properties are experimentally defined, through preparation or measurement. This gives the definition of measurement the appearance of a self-referential cycle, unless we can give an independent definition of preparation. We shall come back to this later in Section 5.

On the other hand, deducing (1) theoretically is a difficult task of statistical mechanics, since the instrument is a macroscopic body that, on the fundamental level necessary for a foundation, can be treated only in terms of statistical mechanics. The investigation of this in Subsections 4.3 and 5.1 will show essential differences between the traditional interpretations and the thermal interpretation.

Taking $P(\rho) = \text{tr } \rho A = \langle A \rangle$, we get as special case the following principle. It defines, in agreement with the general uncertainty principle (GUP) and today's NIST standard for specifying uncertainty (TAYLOR & KUYATT [72]) what it means to have measured a quantity:

(MP) Measurement principle: *A macroscopic quantum device qualifies as an instrument for approximately, with uncertainty Δa , measuring a Hermitian quantity A of a system with density operator ρ , if it satisfies the following two conditions:*

(i) (**uncertainty**) *All measured results a deviate from \bar{A} by approximately Δa . The measurement uncertainty is bounded below by $\Delta a \geq \sigma_A$.*

(ii) (**reproducibility**) *If the measurement can be sufficiently often repeated on systems with the same or a sufficiently similar state then the sample mean of $(a - \bar{A})^2$ approaches Δa^2 .*

As customary, one writes the result of a measurement as an **uncertain number** $a \pm \Delta a$ consisting of the measured value a and its uncertainty deviation Δa , with the meaning that the error $|a - \bar{A}|$ is at most a small multiple of Δa . Because of possible systematic errors, it is generally not possible to interpret a as mean value and Δa as standard deviation. Such an interpretation is valid only if the instrument is calibrated to be unbiased.

The measurement principle (MP) creates the foundation of measurement theory. Physicists doing quantum physics (even those adhering to the shut-up-and-calculate mode of working) use this rule routinely and usually without further justification. The rule applies universally. No probabilistic interpretation is needed. In particular, the first part applies also to single measurements of single systems.

The validity of the measurement principle for a given instrument must either be derivable from quantum models of the instrument by a theoretical analysis, or it must be checkable by experimental evidence by calibration. In general, the theoretical analysis leads to difficult problems in statistical mechanics that can be solved only approximately, and only in idealized situations. From such idealizations one then transfers insight to make educated guesses in cases where an analysis is too difficult, and adjusts parameters in the design of the instrument by an empirical calibration process.

Consistent with the general uncertainty principle (GUP), the measurement principle (MP) demands that any instrument for measuring a quantity A has an uncertainty $\Delta a \geq \sigma_A$.² It is an open problem how to prove this from the statistical mechanics of measurement models. But that such a limit cannot be overcome has been checked in the early days of quantum mechanics by a number of thought experiments. Today it is still consistent with experimental capabilities and no serious proposals exist that could possibly change this situation.

In particular, exact measurements have $\Delta a = 0$ and hence $\sigma_A = 0$. This indeed happens for measurements of systems in a pure state when the state vector is an eigenstate of the quantity measured. Thus part of Born's rule holds: *Whenever a quantity A is measured exactly,³ its value is an eigenvalue of A .* But for inexact (i.e, almost all) measurements, the thermal interpretation rejects Born's rule as an axiom defining what counts as a measurement result. With this move, all criticism from Part I [50, Section 3] becomes void since

²The formulation "at least of the order of σ_A " allows for the frequent situation that the measurement uncertainty is larger than the intrinsic (theoretical) uncertainty σ_A .

³But the discrete measurements in a Stern–Gerlach experiment, say, are not exact measurements in this sense, but very low accuracy measurements of the associated q -expectations; see Subsection 2.5.

Born's rule remains valid only in a limited validity; see Subsection 5.2.

2.2 Statistical and deterministic measurements

The requirement (MP) for a measuring instrument includes the reproducibility of the resulting measurement values. Reproducibility in the general sense that all systems prepared in the same state have to behave alike when measured is a basic requirement for all natural sciences. The term "alike" has two different interpretations depending on the context: Either "alike" is meant in the deterministic sense of "approximately equal within the specified accuracy". Or "alike" is meant in the statistical sense of "approximately reproducing in the long run the same probabilities and mean values". An object deserves the name "instrument" only if it behaves in one or other of these ways.

Corresponding to the two meanings we distinguish two kinds of measuring instruments, deterministic ones and statistical ones. Consequently, the quantitative relationship between the system state and the measurement results may be deterministic or statistical, depending on what is measured.

Radioactive decay, when modeled on the level of individual particles, is a typical **statistical** phenomenon. It needs a stochastic description as a branching process, similar to classical birth and death processes in biological population dynamics. The same holds for particle scattering, the measurement of cross sections, since particles may be created or annihilated, and for detection events, such as recording photons by a photoelectric device or particle tracks in a bubble chamber.

On the other hand, although quantum physics generally counts as an intrinsically probabilistic theory, it is important to realize that it not only makes assertions about probabilities but also makes many **deterministic** predictions verifiable by experiment. These deterministic predictions fall into two classes:

(i) Predictions of numerical values believed to have a precise value in nature:

- The most impressive proof of the correctness of quantum field theory in microphysics is the magnetic moment of the electron, predicted by quantum electrodynamics (QED) to the phenomenal accuracy of 12 significant digit agreement with the experimental value. It is a universal constant, determined solely by the two parameters in QED, the electron mass and the fine structure constant.
- QED also predicts correctly emission and absorption spectra of atoms and molecules, both the spectral positions and the corresponding line widths.
- Quantum hadrodynamics allows the prediction of the masses of all isotopes of the chemical elements in terms of models with only a limited number of parameters.

(ii) Predictions of qualitative properties, or of numerical values believed to be not exactly determined but which are accurate with a tiny, computable uncertainty.

- The modern form of quantum mechanics was discovered through its successful description

and organization of a multitude of spectroscopic details on the position and width of spectral lines in atomic and molecular spectra.

- QED predicts correctly the color of gold, the liquidity of mercury at room temperature, and the hardness of diamond.
- Quantum physics enables the computation of thermodynamic equations of state for a huge number of materials. Equations of states are used in engineering in a deterministic manner, with negligible uncertainty. Engineers usually need not explicitly consider quantum effects since these are encoded in their empirical formulas for the equations of states.
- quantum chemistry predicts correctly rates of chemical reactions.
- From quantum physics one may also compute transport coefficients for deterministic kinetic equations used in a variety of applications.

Thus quantum physics makes both deterministic and statistical assertions, depending on which system it is applied to and on the state or the variables to be determined. Statistical mechanics is mainly concerned with deterministic prediction of class (ii) in the above classification.

Predictions of class (i) are partly related to spectral properties of the Hamiltonian of a quantum system, and partly to properties deduced from form factors, which are deterministic byproducts of scattering calculations. In both cases, classical measurements account adequately for the experimental record.

The traditional interpretations of quantum mechanics do only rudimentarily address the deterministic aspects of quantum mechanics, requiring very idealized assumptions (being in an eigenstate of the quantity measured) that are questionable in all deterministic situations described above.

2.3 Macroscopic systems and deterministic instruments

A **macroscopic system** is a system large enough to be described sufficiently well by the methods of statistical mechanics,⁴ where, due to the law of large numbers, one obtains essentially deterministic results.

The weak law of large numbers implies that quantities averaged over a large population of identically prepared systems become highly significant when their value is nonzero, even when no single quantity is significant. This explains the success of Boltzmann's statistical mechanics to provide an effectively deterministic description of ideal gases, where all particles may be assumed to be independent and identically prepared.

In real, nonideal gases, the independence assumption is only approximately valid because of possible interactions, and in liquids, the independence is completely lost. The power of

⁴However, as discussed by SKLAR [67], both the frequentist and the subjective interpretation of probability in statistical mechanics have significant foundational problems, already in the framework of classical physics. These problems are absent in the thermal interpretation, where single systems are described by mixed states, without any implied statistical connotation.

the statistical mechanics of Gibbs lies in the fact that it allows to replace simple statistical reasoning on populations based on independence by more sophisticated algebraic techniques that give answers even in extremely complex interacting cases. Typically, the uncertainty is of the order $O(N^{-1/2})$, where N is the mean number of identical microsystems making up the macroscopic system. Thus the thermal interpretation associates to macroscopic objects essentially classical quantities whose uncertain value (q-expectation) has a tiny uncertainty only.

In particular, the macroscopic pointer of a measurement instrument always has a well-defined position, given by the q-expectation of the Heisenberg operator $x(t)$ corresponding to the center of mass of its $N \gg 1$ particles at time t . The uncertain pointer position at time t is $\langle x(t) \rangle \pm \sigma_{x(t)}$, where the q-expectation is taken in the Heisenberg state of the universe (or any sufficiently isolated piece of it). Thus the position is fully determined by the state of the pointer – but it is an uncertain position. By the law of large numbers, the uncertainty $\sigma_{x(t)}$ is of order $N^{-1/2}$. Typically, this limit accuracy is much better than the accuracy of the actual reading. Thus we get well-defined pointer readings, leading within the reading accuracy to deterministic measurement results.

Whether by this or by other means, whenever one obtains an essentially deterministic measurement result, we may say that measuring is done by a deterministic instrument:

A **deterministic instrument** is a measuring instrument that measures beables, deterministic functions $F(\rho)$ of the state ρ of the system measured, within some known margin of accuracy, in terms of some property read from the instrument, a macroscopic system. A special case is the measurement of a quantity A , since the uncertain value $\bar{A} = \text{Tr } \rho A$ of A is a function of the state ρ of the system. Thus if measurements yield values $a \approx \bar{A}$ within some uncertainty Δa , the corresponding instrument is a deterministic instrument for **measuring** A within this accuracy.

2.4 Statistical instruments

The measurement of a tiny, **microscopic** system, often consisting of only a single particle, is of a completely different nature. Now the uncertainties do not benefit from the law of large numbers, and the relevant quantities often are no longer significant, in the sense that their uncertain value is already of the order of their uncertainties. In this case, the necessary quantitative relations between properties of the measured system and the values read off from the measuring instrument are only visible as stochastic correlations.

The results of single measurements are no longer reproducibly observable numbers. In the thermal interpretation, a single detection event is therefore not regarded as a measurement of a property of a measured microscopic system, but only as a property of the macroscopic detector correlated to the nature of the incident fields.

This is the essential part where the thermal interpretation differs from tradition. Indeed, from a single detection event, one can only glean very little information about the state of a microscopic system. Conversely, from the state of a microscopic system one can usually predict only probabilities for single detection events.

All readings from a photographic image or from the scale of a measuring instrument, done by an observer, are deterministic measurements of an instrument property by the observer. Indeed, what is measured by the eye is the particle density of blackened silver on a photographic plate or that of iron of the tip of the pointer on the scale, and these are extensive variables in a continuum mechanical local equilibrium description of the instrument.

The historically unquestioned interpretation of such detection events as the measurement of a particle position is one of the reasons for the failure of traditional interpretations to give a satisfying solution of the measurement problem. The thermal interpretation is here more careful and treats detection events instead as a statistical measurement of particle beam intensity.

To obtain comprehensive information about the state of a single microscopic system is therefore impossible. To collect enough information about the prepared state and hence the state of a system measured, one needs either time-resolved measurements on a single stationary system (available, e.g., for atoms in ion traps or for electrons in quantum dots), or a population of identically prepared systems. In the latter case, one can get useful microscopic state information through quantum tomography, cf. Subsection 2.5.

Thus in case of measurements on microscopic quantum systems, the quantitative relationship between measurement results and measured properties only takes the form of a statistical correlation. The reproducibly observable items, and hence the carrier of scientific information, are statistical mean values and probabilities. These are indeed predictable by quantum physics. But – in contrast to the conventional terminology applied to single detection events for photons or electrons – *the individual events no longer count as definite measurements of single system properties.*

This characteristics of the thermal interpretation is an essential difference to traditional interpretations, for which each event is a definite measurement.

A **statistical instrument** determines its final measurement results from a large number of raw measurements by averaging or by more advanced statistical procedures, often involving computer processing. Again, due to the law of large numbers, one obtains essentially deterministic results, but now from very noisy raw measurements. Examples include low intensity photodetection, the estimation of probabilities for classical or quantum stochastic processes, astronomical instruments for measuring the properties of galaxies, or the measurement of population dynamics in biology.

This behaviour guarantees reproducibility. In other words, systems prepared in the same state behave in the same way under measurement – in a deterministic sense for a deterministic instrument, and in a statistical sense for a statistical one. In both cases, the final measurement results approximate with a limited accuracy the value of a function F of the state of the system under consideration.

2.5 Event-based measurements

Measurements in the form of discrete events (such as the appearance of clicks, flashes, or particle tracks) may be described in terms of an **event-based instrument** characterized by a discrete family of possible measurement results a_1, a_2, \dots that may be real or complex numbers, vectors, or fields, and nonnegative Hermitian quantities P_1, P_2, \dots satisfying

$$P_1 + P_2 + \dots = 1. \quad (2)$$

The nonnegativity of the P_k implies that all q-probabilities

$$p_k = \langle P_k \rangle = \text{Tr } \rho P_k \quad (3)$$

are nonnegative, and (2) guarantees that the q-probabilities always add up to 1. By its definition, the notion of q-probabilities belongs to the formal core of quantum mechanics and is independent of any interpretation.

Unlike in all traditional interpretations, the thermal interpretation considers the observable result a_k not as exact measurement results of some "observable" with counterintuitive quantum properties but as a (due to the tiny sample size very low accuracy) statistical measurements of certain q-expectations.

In the thermal interpretation all q-expectations are beables; in particular, all q-probabilities are among the beables. As described in Part II [51, Subsection 3.5], a q-probability p may be approximately measured as relative frequency, whenever there is an event-generating device (the **preparation**) that produces a large number N of independent copies (realizations) of the same quantum system. In this case, we require that if the measured system is in the state ρ , the instrument gives the observable result a_k with a relative frequency approaching the q-probability p_k as the sample size gets arbitrarily large.

An **event-based instrument** is a statistical instrument measuring the probability of events modeled by a discrete (classical or quantum) statistical process. In the quantum case, it is mathematically described by a **positive operator-valued measure**, short **POVM**, defined as a family P_1, P_2, \dots of Hermitian, positive semidefinite operators satisfying (2) (or a continuous generalization of this).

POVMs originated around 1975 in work by HELSTROM [36] on quantum detection and estimation theory and are discussed in some detail in PERES [56]. They describe the most general quantum measurement of interest in quantum information theory. Which operators P_k correctly describe a statistical instrument can in principle be found out by suitable **calibration measurements**. Indeed, if we feed the instrument with enough systems prepared in known states ρ_j , we can measure approximate probabilities $p_{jk} \approx \langle P_k \rangle_j = \text{Tr } \rho_j P_k$. By choosing the states diverse enough, one may approximately reconstruct P_k from this information by a process called **quantum tomography**. In quantum information theory, the Hilbert spaces are finite-dimensional, hence the quantities form the algebra $\mathbb{E} = \mathbb{C}^{N \times N}$ of complex $N \times N$ matrices. In this case, the density operator is **density matrix** ρ , a complex Hermitian $N \times N$ -matrix with trace one, together with the trace formula

$$\langle A \rangle = \text{Tr } \rho A.$$

Since $\langle 1 \rangle = 1$, a set of $N^2 - 1$ binary tests for specific states, repeated often enough, suffices for the state determination. Indeed, it is easy to see that repeated tests for the states e^j , the unit vectors with just one entry one and other entries zero, tests the diagonal elements of the density matrix, and since the trace is one, one of these diagonal elements can be computed from the knowledge of all others. Tests for $e^j + e^k$ and $e^j + ie^k$ for all $j < k$ then allow the determination of the (j, k) and (k, j) entries. Thus frequent repetition of a total of $N - 1 + 2\binom{N}{2} = N^2 - 1$ particular tests determines the full state. The optimal reconstruction to a given accuracy, using a minimal number of individual measurements, is the subject of **quantum estimation theory**, still an active frontier of research.

Distinguished from a stochastic instrument performing event-based measurements is an **event-based filter**, which turns an input state ρ with probability

$$p_k := \langle R_k^* R_k \rangle$$

into an output state

$$\rho_k := \frac{1}{p_k} R_k \rho R_k^*.$$

Here the R_k are operators satisfying

$$\sum_k R_k^* R_k = 1.$$

Which case occurred may be considered as an event; the collection of possible events is then described by the POVM with $P_k := R_k^* R_k$.

2.6 The thermal interpretation of eigenvalues

As discussed already in Part I [50], the correspondence between observed values and eigenvalues is only approximate, and the quality of the approximation improves with reduced uncertainty. The correspondence is perfect only at zero uncertainty, i.e., for completely sharp observed values. To discuss this in detail, we need some results from functional analysis. The **spectrum** $\text{Spec } A$ of a linear operator on a Euclidean space \mathbb{H} (a common domain of all relevant q-observables of a system) is the set of all $\lambda \in \mathbb{C}$ for which no linear operator $R(\lambda)$ from the completion $\overline{\mathbb{H}}$ of \mathbb{H} to \mathbb{H} exists such that $(\lambda - A)R(\lambda)$ is the identity. $\text{Spec } A$ is always a closed set.

A linear operator $A \in \text{Lin } \mathbb{H}$ is called **essentially self-adjoint** if it is Hermitian and its spectrum is real (i.e., a subset of \mathbb{R}). For N -level systems, where \mathbb{H} is finite-dimensional, the spectrum coincides with the set of eigenvalues, and every Hermitian operator is essentially self-adjoint. In infinite dimensions, the spectrum contains the eigenvalues, but not every number in the spectrum must be an eigenvalue; and whether a Hermitian operator is essentially self-adjoint is a question of correct boundary conditions.

Theorem. Let A be essentially self-adjoint, with value $\overline{A} := \langle A \rangle$ and q-standard deviation σ_A in a given state. Then the spectrum of A contains some real number λ with

$$|\lambda - \overline{A}| \leq \sigma_A. \quad (4)$$

Proof. The linear operator $B = (A - \bar{A})^2 - \sigma_A^2$ is a quadratic function of A , hence its spectrum consists of all $\lambda' := (\lambda - \bar{A})^2 - \sigma_A^2$ with $\lambda \in \text{Spec } A$; in particular, it is real. Put $\lambda_0 := \inf \text{Spec } B$. Then $B - \lambda_0$ is a Hermitian operator with a real, nonnegative spectrum, hence positive semidefinite. (In infinite dimensions, this requires the use of the spectral theorem.) Thus $B - \lambda_0 \geq 0$ and $0 \leq \langle B - \lambda_0 \rangle = \langle (A - \bar{A})^2 \rangle - \sigma_A^2 - \lambda_0 = -\lambda_0$. Therefore $\lambda_0 \leq 0$. Since $\text{Spec } B$ is closed, λ_0 is in the spectrum, hence has the form $(\lambda - \bar{A})^2 - \sigma_A^2$ with $\lambda \in \text{Spec } A$. This λ satisfies (4). \square

In particular, if, in some state, A has a **sharp** observable value, defined by $\sigma_A = 0$, then the value $\langle A \rangle$ belongs to the spectrum. In practice, this is the case only for quantities A whose spectrum (set of sharp values) consists of rationals with small numerator and denominator. Examples are spin and polarization in a given direction, (small) angular momentum, and (small) particle numbers.

3 Particles from quantum fields

In continuation of the discussion in Subsection 4.4 of Part I [50], we discuss in this Section the extent to which a particle picture of matter and radiation is appropriate.

In physics practice, it is often unavoidable to switch between representations featuring different levels of detail. The fundamental theory of elementary particles and fields, with the most detailed description, is quantum field theory. Since quantum field theory is fundamental, the simpler quantum mechanics of particles is necessarily a derived description.

How to obtain the quantum mechanics of particles from relativistic interacting quantum field theory is a nontrivial problem. The traditional textbook description in terms of scattering and associated propagators does not give a description at finite times.

In the fundamental reality – i.e., represented by beables of quantum field theory, expressed at finite times in hydrodynamic terms –, fields concentrated in fairly narrow regions move along uncertain flow lines determined by effective field equations.

In the particle description, these fields are somehow replaced by a quantum mechanical model of moving particles. The uncertainty is now accounted for by the uncertain value of the position $\mathbf{q}(t)$ of each particle together with its uncertainty $\sigma_{\mathbf{q}(t)}$, at any time t , providing not a continuous trajectory but a fuzzy world tube defining their location. The momentum of the quantum particles is also uncertain. For example, the momentum vector of a particle at CERN is measured by collecting information from many responding wires and applying curve fitting techniques to get an approximate curve of positions at all times and inferring from its derivative an uncertain momentum. Similar techniques are used for particle tracks on photographic plates or in bubble chambers.

How one finds from a relativistic quantum field description of a beam a corresponding quantum mechanical particle description has hardly received attention so far. While informally, particles are considered to be elementary excitations of the quantum fields, this can

be given an exact meaning only for free field theories. In interacting relativistic quantum fields, the notion is, at finite times, approximate only.

That the approximation problem is nontrivial can be seen from the fact that in quantum field theory, position is a certain parameter. whereas in the quantum mechanics of particles, position is an uncertain quantity. Thus in the approximation process, position loses its parameter status and becomes uncertain. How, precisely, is unknown.

3.1 Fock space and particle description

A precise correspondence between particles and fields is possible only in free quantum field theories. These are described by distribution-valued operators on a Fock space. The latter is completely determined by its 1-particle sector, the single particle space.

Poincare invariance, locality, and the uniqueness of the vacuum state imply that the single particle space of a free quantum field theory furnishes a causal unitary irreducible representation of the Poincare group. These representations were classified in 1939 by WIGNER [77]. This is why particle theorists say that elementary particles are causal unitary irreducible representations of the Poincare group, Thus elementary particles are something exceedingly abstract, not tiny, fuzzy quantum balls!

For spin ≤ 1 , these representations happen to roughly match the solution space of certain wave equations for a single relativistic particle in the conventional sense of quantum mechanics, but only if one discards the contributions of all negative energy states of the latter. In relativistic quantum field theory, the latter reappear as states for antiparticles – a different kind of particles with different properties. This already shows that there is something very unnatural about the relativistic particle picture on the quantum-mechanical single-particle level.

In general, a field description on the particle level in terms of a conventional multiparticle structure is necessarily based on a Fock space representation with a number operator N with spectrum consisting precisely of the nonnegative integers. The eigenspace for the eigenvalue 1 of N then defines the bare single-particle Hilbert space. In the relativistic case, the resulting description is one in terms of bare, unphysical particles.

Untangling the S-matrix using bare perturbation theory replaces the real-time dynamics of the quantum fields by an non-temporal infinite sum of contributions of multivariate integrals depicted in shorthand by Feynman diagrams showing a web of virtual particles. The Feynman diagrams provide a pictorial representation of the formalism of bare perturbation theory. Free real particles show as external lines, while the interaction is represented in terms of internal lines, figuratively called virtual particles. Most of the resulting integrals (all except the tree diagrams) are infinite and physically meaningless. A renormalization process turns the sum of all diagrams with a fixed number of loops (where the infinities cancel) into finite numbers whose sum over not too high orders (the series is asymptotic only) has an (approximate) physical meaning. But in the renormalization process the intuitive connection of the lines depicted in Feynman diagrams – the alleged world lines of virtual particles, in the popular myth (cf. NEUMAIER [52]) – gets completely lost. Nothing

resembles anything like a process in time – described by the theory and the computations is only a black box probabilistic model of the in-out behavior of multiparticle scattering.

3.2 Physical particles in interacting field theories

All our knowledge concerning the internal properties of atoms is derived from experiments on their radiation or collision reactions, such that the interpretation of experimental facts ultimately depends on the abstractions of radiation in free space, and free material particles. [...] The use of observations concerning the behaviour of particles in the atom rests on the possibility of neglecting, during the process of observation, the interaction between the particles, thus regarding them as free. [...]

The wave mechanical solutions can be visualised only in so far as they can be described with the aid of the concept of free particles. [...]

Summarising, it might be said that the concepts of stationary states and individual transition processes within their proper field of application possess just as much or as little 'reality' as the very idea of individual particles.

Niels Bohr, 1927 [8, pp.586–589]

While the conventional construction of relativistic quantum field theories starts with Fock space, a relativistic interacting quantum field itself cannot be described in terms of a Fock space. The Fock space structure of the initial scaffolding is destroyed by the necessary renormalization, since the number operator cannot be renormalized. Only the asymptotic fields figuring in the S-matrix reside in a Fock space – for colored quarks because of confinement not even in a conventional Fock space with a positive definite inner product, but only in an indefinite Fock–Krein space.

As a consequence, the particle concept is only asymptotically valid, under conditions where particles are essentially free. Traditionally, the discussion of particle issues in relativistic interacting quantum fields is therefore restricted to scattering processes involving asymptotic particle states. Only the S-matrix provides meaning to quantum particles, in an asymptotic sense, describing Born's rule for scattering processes. In the formulation of Part I [50, Subsection 3.1]: In a scattering experiment described by the S-matrix S ,

$$\Pr(\psi_{\text{out}}|\psi_{\text{in}}) := |\psi_{\text{out}}^* S \psi_{\text{in}}|^2$$

is the conditional probability density that scattering of particles prepared in the in-state ψ_{in} results in particles in the out-state ψ_{out} .

Indeed, textbook scattering theory for elementary particles is the *only* place where Born's rule is used in quantum field theory. Here the in- and out-states are asymptotic eigenstates of total momentum, labelled by a maximal collection of independent quantum numbers (including particle momenta and spins). An **asymptotic quantity** is a q-observable still visible in the limits of time $t \rightarrow \infty$ or $t \rightarrow -\infty$, so that scattering theory says something interesting about it. This is relevant since quantum dynamics is very fast but measurements take time. Measuring times are already very well approximated by infinity, on the time

scale of typical quantum processes. Thus only asymptotic quantities have a reasonably well-defined response. That's why information about microsystems is always collected via scattering experiments described by the S-matrix, which connects asymptotic preparation at time $t = -\infty$ with asymptotic measurement at time $t = +\infty$. Particle momenta (like other conserved additive quantities) are asymptotic quantities.

In quantum field theory, scattering theory is just the special case of a universe containing only a tiny number of particles with known momentum at time $t = -\infty$, whose behavior at time $t = +\infty$ is to be predicted. This caricature of a universe is justified only when the few-particle system is reasonably well isolated from the remainder of the universe. In a real experiment, this is a good approximation to a collision experiment when the length and time scale of a collision is tiny compared to the length and time scale of the surrounding preparation and detection process. Much care is taken in modern colliders to achieve this to the required degree of accuracy.

3.3 Semiclassical approximation and geometric optics

In the preceding, we discussed the precise notion of particles in relativistic quantum field theory – an asymptotic notion only. Cross sections for the scattering processes computed in this way are supposed to be exact (assuming the idealization that the underlying theory is exact and the computations are done exactly).

However, the particle picture has another very practical use, as an approximate, semiclassical concept valid whenever the fields are concentrated along a single (possibly bent) ray and the resolution is coarse enough. When these conditions apply, one is no longer in the full quantum domain and can already describe everything semiclassically, i.e., classical with small quantum corrections. Thus the particle concept is useful when and only when the semiclassical description is already adequate. Whenever one uses the particle picture beyond scattering theory (and in particular always when one has to interpret what people using the particle language say), one silently acknowledges that one works in a semiclassical picture where a particle description makes approximate sense except during collisions.

A **particle** is a blob of high field concentrations well-localized in phase space (i.e., in the kinetic approximation of quantum field theory), with a boundary whose width (or the width in transversal directions for a moving particle) is tiny compared to its diameter.

Thus field concentrations must be such that their (smeared) density peaks at reasonably well-defined locations in phase space. At this point, similar to the regime of geometric optics for classical electromagnetic fields these peaks behave like particles. Thus particles are approximately defined as local excitations of a field, and they have (as wavelets in classical mechanics) an uncertain (not exactly definable) position. Their (necessarily approximate) position and momentum behaves approximately classically (and gives rise to a classical picture of quantum particles) in the regime corresponding to geometric optics. When the spatial resolution is such that the conditions for the applicability of geometric optics hold, particles can be used as an adequate approximate concept.

In a collision experiment, it is valid to say that particles travel on incoming and outgoing

beams in spacetime while they are far apart, since this is a good semiclassical description of the free particles in a paraxial approximation. But when they come close, the semiclassical description breaks down and one needs full quantum field theory to describe what happens.

The exact state of the interacting system is now a complicated state in a renormalized quantum field Hilbert space⁵ that no one so far was able to characterize; it is only known (Haag's theorem) that it cannot be the asymptotic Fock space describing the noninteracting particles. Since it is not a Fock space, talking about particles during the interaction makes no longer sense - the quantum fields of which the particles are elementary excitations become very non-particle like. After the collision products separate well enough, the semiclassical description becomes feasible again, and one can talk again about particles traveling along beams.

Thus while the field picture is always valid, the picture of particles traveling along beams or other world tubes is appropriate except close to the collision of two world tubes. The behavior there is effectively described in a black box fashion by the S-matrix. This is a reasonable approximation if the collision speed is high enough, so that one can take the in- and outgoing particles as being at time $-\infty$ and $+\infty$, and can ignore what happens at finite times, i.e., during the encounter. Thus, in the **semiclassical** description, we have between collisions real particles described by asymptotic states, while the collisions themselves – where the particle picture no longer make sense – are described using a black box view featuring the S-matrix, To calculate the S-matrix one may work in renormalized perturbation theory using quantum field theory.

Using the intuition of geometric optics requires a locally free effective description. In a locally homogeneous background, such an effective description is usually achievable through the introduction of **quasiparticles**. These are collective field modes that propagate as if they were free. If the composition of the background changes, the definition of the quasiparticles changes as well.

In particular, the photons in glass or air are quasiparticles conceptually different from those in vacuum. Similarly, the moving electrons in a metal are quasiparticles conceptually different from those in vacuum. This shows that photons, electrons, and other elementary particles have no conceptual identity across interfaces. A photon, traditionally taken to be emitted by a source, then passing a system of lenses, prisms, half-silvered mirrors, and other optical equipment, changes its identity each time it changes its environment!

This is corroborated by the field of **electron optics**, where geometric rays are used to calculate properties of magnetic and electrostatic lenses for electron beams.

Problems abound if one tries to push the analogies beyond the semiclassical domain of validity of the particle concept. Already in classical relativistic mechanics, point trajectories are idealizations, restricted to a treatment of the motion of a single point in a classical external field. By a result of CURRIE et al. [19], classical relativistic multi-particle point trajectories are inconsistent with a Hamiltonian dynamics. Thus one should not expect

⁵Because of superselection sectors, this Hilbert space is generally nonseparable, a direct sum of the Hilbert spaces corresponding to the different superselection sectors.

them to exist in quantum physics either. They are appropriate only as an approximate description.

Note that this semiclassical domain of validity of the particle picture excludes experiments with multilocal fields generated by beam-splitters, half-silvered mirrors, double slits, diffraction, long-distance entanglement, and the like. It is there where the attempt to stick to the particle picture leads to all sorts of counterintuitive features. But these are caused by the now inadequate particle imagery, not by strange features of quantum field theory itself.

3.4 The photoelectric effect

In quantum optics experiments, both sources and beams are extended macroscopic objects describable by quantum field theory and statistical mechanics, and hence have (according to the thermal interpretation) associated nearly classical observables – densities, intensities, correlation functions – computable from quantum physics in terms of q-expectations.

An instructive example is the **photoelectric effect**, the measurement of a classical free electromagnetic field by means of a photomultiplier. A detailed discussion is given in Sections 9.1–9.5 of MANDEL & WOLF [47]; here we only give an informal summary of their account.

Classical input to a quantum system is conventionally represented in the Hamiltonian of the quantum system by an interaction term containing the classical source as an external field or potential. In the semiclassical analysis of the photoelectric effect, the detector is modeled as a many-electron quantum system, while the incident light triggering the detector is modeled as an external electromagnetic field. The result of the analysis is that if the classical field consists of electromagnetic waves (light) with a frequency exceeding some threshold then the detector emits a random stream of photoelectrons with a rate that, for not too strong light, is proportional to the intensity of the incident light. The predictions are quantitatively correct for normal light.

The response of the detector to the light is statistical, and only the rate (a short time mean) with which the electrons are emitted bears a quantitative relation with the intensity. Thus the emitted photoelectrons form a statistical measurement of the intensity of the incident light.

The results on this analysis are somewhat surprising: Although the semiclassical model used to derive the quantitatively correct predictions does not involve photons at all, the discrete nature of the electron emissions implies that a photodetector responds to classical light as if it were composed of randomly arriving photons! (The latter was the basis for the original explanation of the photoeffect for which Einstein received the Nobel prize.)

This proves that the discrete response of a photodetector cannot be due to the quantum nature of the detected object.

The classical external field discussed so far is of course only an approximation to the quantum electromagnetic field, and was only used to show that the discrete response of a photodetector cannot be due to its interactions with particles, or more generally not to the

quantum nature of the detected object. The discrete response is due to the detector itself, and triggered by the interaction with a field. A field mediating the interaction must be present with sufficient intensity to transmit the energy necessary for the detection events. Both a classical and a quantum field produce such a response. Only the quantitative details change in the case of quantum fields, but nothing depends on the presence or absence of "photons". Thus photons are figurative properties of quantum fields manifesting themselves only in the detectors. Before detection, there are no photons; one just has beams of light in an entangled state.

This shows the importance of differentiating between prepared states of the system (here of classical or quantum light) and measured events in the instrument (here the amplified emitted electrons). The measurement results are primarily a property of the instrument, and their interpretation as a property of the system measured needs theoretical analysis to be conclusive.

3.5 A classical view of the qubit

It is commonly said that quantum mechanics originated in 1900 with Max Planck, reached its modern form with Werner Heisenberg and Erwin Schrödinger, got its correct interpretation with Max Born, and its modern mathematical formulation with Paul Dirac and John von Neumann. It is very little known that much earlier – in 1852, at a time when Planck, Heisenberg, Schrödinger, Born, Dirac, and von Neumann were not even born –, George Stokes described all the modern quantum phenomena of a single qubit, explaining them in classical terms.

Remarkably, this description of a qubit is fully consistent with the thermal interpretation of quantum physics. Stokes' description is couched in the language of optics – polarized light was the only quantum system that, at that time, was both accessible to experiment and quantitatively understood. Stokes' classical observables are the functions of the components of the coherence matrix, the optical analogue of the density operator of a qubit, just as the thermal interpretation asserts.

The transformation behavior of rays of completely polarized light was first described in 1809 by Etienne-Louis MALUS [46] (who coined the name "polarization"); that of partially polarized light in 1852 by George STOKES [70]. This subsection gives a modern description of the core of this work by Malus and Stokes.

We shall see that Stokes' description of a polarized quasimonochromatic beam of classical light behaves exactly like a modern quantum bit.

A ray (quasimonochromatic beam) of polarized light of fixed frequency is characterized by a state, described equivalently by a real **Stokes vector**

$$S = (S_0, S_1, S_2, S_3)^T = \begin{pmatrix} S_0 \\ \mathbf{S} \end{pmatrix}$$

with

$$S_0 \geq |\mathbf{S}| = \sqrt{S_1^2 + S_2^2 + S_3^2},$$

or by a **coherence matrix**, a complex positive semidefinite 2×2 matrix ρ . These are related by

$$\rho = \frac{1}{2}(S_0 + \mathbf{S} \cdot \boldsymbol{\sigma}) = \frac{1}{2} \begin{pmatrix} S_0 + S_3 & S_1 - iS_2 \\ S_1 + iS_2 & S_0 - S_3 \end{pmatrix},$$

where $\boldsymbol{\sigma}$ is the vector of Pauli matrices. $\text{Tr} \rho = S_0$ is the **intensity** of the beam. $p = |\mathbf{S}|/S_0 \in [0, 1]$ is the **degree of polarization**. Note the slight difference to density matrices, where the trace is required to be one.

A linear, non-mixing (not depolarizing) instrument (for example a polarizer or phase rotator) is characterized by a complex 2×2 **Jones matrix** T . The instrument transforms an in-going beam in the state ρ into an out-going beam in the state $\rho' = T\rho T^*$. The intensity of a beam after passing the instrument is $S'_0 = \text{Tr} \rho' = \text{Tr} T\rho T^* = \text{Tr} \rho T^*T$. If the instrument is lossless, the intensities of the in-going and the out-going beam are identical. This is the case if and only if the Jones matrix T is unitary.

Since $\det \rho = (S_0^2 - S_3^2) - (S_1^2 + S_2^2) = S_0^2 - \mathbf{S}^2$, the fully polarized case $p = 1$, i.e., $S_0 = |\mathbf{S}|$, is equivalent with $\det \rho = 0$, hence holds iff the rank of ρ is 0 or 1. In this case, the coherence matrix can be written in the form $\rho = \psi\psi^*$ with a state vector ψ determined up to a phase. Thus precisely the pure states are fully polarized. In this case, the intensity of the beam is

$$S_0 = \langle 1 \rangle = |\psi|^2 = \psi^*\psi.$$

A **polarizer** has $T = \phi\phi^*$, where $|\phi|^2 = 1$. It reduces the intensity to

$$S'_0 = \langle T^*T \rangle = |\phi^*\psi|^2.$$

This is **Malus' law**.

An instrument with Jones matrix T transforms a beam in the pure state ψ into a beam in the pure state $\psi' = T\psi$. Passage through inhomogeneous media can be modeled by means of many slices consisting of very thin instruments with Jones matrices $T(t)$ close to the identity. If $\psi(t)$ denotes the pure state at time t then $\psi(t + \Delta t) = T(t)\psi(t)$, so that for small Δt (the time needed to pass through one slice),

$$\frac{d}{dt}\psi(t) = \frac{\psi(t + \Delta t) - \psi(t)}{\Delta t} + O(\Delta t) = \frac{(T(t) - 1)}{\Delta t}\psi(t) + O(\Delta t).$$

In a continuum limit $\Delta t \rightarrow 0$ we obtain the time-dependent **Schrödinger equation**

$$i\hbar \frac{d}{dt}\psi(t) = H(t)\psi(t),$$

where (note that $T(t)$ depends on Δt)

$$H(t) = \lim_{\Delta t \rightarrow 0} i\hbar \frac{T(t) - 1}{\Delta t}$$

plays the role of a time-dependent Hamiltonian. Note that in the lossless case, $T(t)$ is unitary, hence $H(t)$ is Hermitian.

A linear, mixing (depolarizing) instrument transforms ρ instead into a sum of several terms of the form $T\rho T^*$. It is therefore described by a real 4×4 **Mueller matrix** acting on the Stokes vector. Equivalently, it is described by a completely positive linear map on the space of 2×2 matrices, acting on the polarization matrix.

Thus we see that a polarized quasimonochromatic beam of classical light behaves exactly like a modern quantum bit. We might say that classical optics is just the quantum physics of a single qubit passing through a medium!

Indeed, the 1852 paper by STOKES [70] described all the modern quantum phenomena for qubits, explained in classical terms. In particular,

- Splitting fully polarized beams into two such beams with different, but orthogonal polarization corresponds to writing a wave function as superposition of preferred basis vectors.
- Mixed states are defined (in his paragraph 9) as arising from "groups of independent polarized streams" and give rise to partially polarized beams.
- The coherence matrix is represented by Stokes with four real parameters, in today's terms comprising the Stokes vector.
- Stokes asserts (in his paragraph 16) the impossibility of recovering from a mixture of several distinct pure states any information about these states beyond what is encoded in the Stokes vector (equivalently, the coherence matrix).
- The latter can be linearly decomposed in many essentially distinct ways into a sum of pure states, but all these decompositions are optically indistinguishable, hence have no physical meaning.

The only difference to the modern description is that the microscopic view is missing. For faint light, photodetection leads to discrete detection events – even in models with an external *classical* electromagnetic field; cf. the discussion in Subsection 3 below. The trace of ρ is the intensity of the beam, and the rate of detection events is proportional to it. After normalization to unit intensity, ρ becomes the density operator corresponding to a single detection event (aka photon).

This is a simple instance of the transition from a beam (classical optics or quantum field) description to a single particle (quantum mechanical) description.

It took 75 years after Stokes until the qubit made its next appearance in the literature, in a much less comprehensive way. In 1927, WEYL [75, pp.8-9] discusses qubits in the guise of an ensemble ("Schwarm") of spinning electrons. Instead of the language of Stokes, the description uses the paradoxical language still in use today, where the meaning of everything must be redefined to give at least the appearance of making sense.

In its modern formulation via Maxwell's equations, classical partially polarized light (as described by Stokes) already requires the stochastic form of these equations, featuring – just like the full quantum description – field expectations and correlation functions; see MANDEL & WOLF [47]. The coherence matrices turn into simple camatrix-valued field correlation functions.

4 The thermal interpretation of statistical mechanics

Like quantum mechanics, quantum statistical mechanics also consists of a formal core and its interpretation. Almost everything done in the subject belongs to the formal core, the formal shut-up-and-calculate part of statistical mechanics, without caring about the meaning of the computed q -expectations. The interpretation is considered to be almost obvious and hence gets very little attention. For example, the well-known statistical physics book by LANDAU & LIFSHITZ [43] dedicates just 7 (of over 500) pages (in Section 5) to the properties of the density operator, the basic object in quantum statistical mechanics, and less than half of these pages concern its interpretation in terms of pure states. Fortunately, no use at all is made of this elsewhere in their book, since, as already discussed in Subsection 3.4 of Part I [50], the "derivation" given there – though one of the most carefully argued available in the literature – is highly deficient. On the other hand, in their thermodynamic implications later in the book, they silently assume the thermal interpretation, by identifying (e.g., in Section 35, where they discuss the grand canonical ensemble) the thermodynamic energy and thermodynamic particle number with the q -expectation of the Hamiltonian and the number operator!

The thermal interpretation revises the interpretation of quantum statistical mechanics and extends this revised interpretation to the microscopic regime, thus accounting for the fact that there is no clear boundary where the macroscopic becomes microscopic. Thus we do not need to assume anything special about the microscopic regime.

Subsection 4.1 shows in which sense classical statistical mechanics is a special case of quantum statistical mechanics; thus it suffices to discuss the quantum case. All statistical mechanics is based on the concept of coarse-graining, introduced in Subsection 4.2. Due to the neglect of high frequency details, coarse-graining leads to stochastic features, either in the models themselves, or in the relation between models and reality. Deterministic coarse-grained models are usually chaotic, introducing a second source of randomness, discussed in Subsection 4.3.

Statistical mechanics proper starts with the discussion of Gibbs states (Subsection 4.4) and the statistical thermodynamics of equilibrium and nonequilibrium (Subsection 4.5). Other ways of coarse-graining lead to quantum-classical models (Subsections 4.6 and 4.7), generating among others the Born–Oppenheimer approximation widely used in quantum chemistry.

4.1 Koopman's representation of classical statistical mechanics

Classical mechanics can be written in a form that looks like quantum mechanics. Such a form was worked out by KOOPMAN [40] for classical statistical mechanics. In the special case where one restricts the expectation mapping to be a $*$ -algebra homomorphism, all uncertainties vanish, and the Koopman representation describes deterministic classical Hamiltonian mechanics.

We discuss classical statistical mechanics in terms of a commutative Euclidean $*$ -algebra

\mathbb{E} of **random variables**, i.e., Borel measurable complex-valued functions on a Hausdorff space Ω , where bounded continuous functions are strongly integrable and the integral is given by $\int f := \int d\mu(X) f(X)$ for some distinguished measure μ . (For a rigorous treatment see NEUMAIER & WESTRA [53].) The quantities and the density operator ρ are represented by multiplication operators in some Hilbert space of functions on phase space. The classical Hamiltonian $H(p, q)$ is replaced by the **Koopman Hamiltonian**

$$\hat{H} := \frac{\partial H(p, q)}{\partial q} i \frac{\partial}{\partial p} - \frac{\partial H(p, q)}{\partial p} i \frac{\partial}{\partial q}.$$

Then both in classical and in quantum statistical mechanics, the state is a density operator. The only difference between the classical and the quantum case is that in the former case, all operators are diagonal. In particular, the classical statistical mechanics of macroscopic matter is also described by (diagonal) Gibbs states.

As discussed in Part II [51], functions of expectations satisfy a Hamiltonian dynamics given by a Poisson bracket. It is not difficult to show that the Koopman dynamics resulting in this way from the Koopman Hamiltonian exactly reproduces the classical Hamiltonian dynamics of arbitrary systems in which the initial condition is treated stochastically. The Koopman dynamics is – like von Neumann’s dynamics – strictly linear in the density matrix. But the resulting dynamics is highly nonlinear when rewritten as a classical stochastic process. This is a paradigmatic example for how nonlinearities can naturally arise from a purely linear dynamics.

Because of the Koopman representation, everything said in the following about quantum statistical mechanics applies as well to classical statistical mechanics.

4.2 Coarse-graining

Die vorher scheinbar unlösbaren Paradoxien der Quantentheorie beruhen alle darauf, daß man diese mit jeder Beobachtung notwendig verbundene Störung vernachlässigt hatte

Werner Heisenberg, 1929 [34, p.495]

The same system can be studied at different levels of resolution. When we model a dynamical system classically at high enough resolution, it must be modeled stochastically since the quantum uncertainties must be taken into account. But at a lower resolution, one can often neglect the stochastic part and the system becomes deterministic. If it were not so, we could not use any deterministic model at all in physics but we often do, with excellent success.

Coarse-graining explains the gradual emergence of classicality, due to the law of large numbers to an ever increasing accuracy as the object size grows. The quantum dynamics changes gradually into classical dynamics. The most typical path is through nonequilibrium thermodynamics (cf. Subsection 4.5 below). There are also intermediate stages modeled by quantum-classical dynamics (see Subsection 4.6 below); these are used in situations where the quantum regime is important for some degrees of freedom but not for others. In fact,

there is a wide spectrum of models leading from full quantum models over various coarse-grained models to models with a fully classical dynamics. One typically selects from this spectrum the model that is most tractable computationally given a desired accuracy.

A coarse-grained model is generally determined by singling out a vector space R of **relevant quantities** whose q -expectations are the variables in the coarse-grained model. If the coarse-grained model is sensible one can describe a deterministic or stochastic **reduced dynamics** of these variables alone, ignoring all the other q -expectations that enter the deterministic Ehrenfest dynamics (see Part II [51, Subsection 2.1]) of the detailed description of the system. These other variables therefore become **hidden variables** that would determine the stochastic elements in the reduced stochastic description, or the prediction errors in the reduced deterministic description. The hidden variables describe the unmodeled **environment** associated with the reduced description.⁶ Note that the same situation in the reduced description corresponds to a multitude of situations of the detailed description, hence each of its realizations belongs to different values of the hidden variables (the q -expectations in the environment), slightly causing the realizations to differ. Thus any coarse-graining results in small prediction errors, which usually consist of neglecting experimentally inaccessible high frequency effects. These uncontrollable errors are induced by the variables hidden in the environment and introduce a stochastic element in the relation to experiment even when the coarse-grained description is deterministic.

The thermal interpretation claims that this influences the results enough to cause all randomness in quantum physics, so that there is no need for intrinsic probability as in traditional interpretations of quantum mechanics. In particular, it should be sufficient to explain from the dynamics of the universe the statistical features of scattering processes and the temporal instability of unobserved superpositions of pure states – as caused by the neglect of the environment.

To give a concrete example of coarse-graining we mention JEON & YAFFE [38], who derive the hydrodynamic equations from quantum field theory for a real scalar field with cubic and quartic self-interactions. Implicitly, the thermal interpretation is used, which allows them to identify field expectations with the classical values of the field.

There are many systems of practical interest where the most slowly varying degrees of freedom are treated classically, whereas the most rapidly oscillating ones are treated in a quantum way. The resulting quantum-classical dynamics, discussed in Subsection 4.6 below, also constitutes a form of coarse-graining. The approximation of fields (with an infinite number of degrees of freedom) by finitely many particles is also a form of coarse-graining.

In the context of coarse-graining models given in a Hamiltonian quantum framework, the **Dirac–Frenkel variational principle** may be profitably used for coarse-graining whenever a pure state approximation is reasonable. This principle is based on the fact that the

⁶They may be regarded as the hidden variables for which Einstein and others searched for so long. Most of them are highly non-local, in accordance with Bell’s theorem. The thermal interpretation thus reinstates nonlocal hidden variable realism, but – unlike traditional hidden variable approaches – without introducing additional degrees of freedom into quantum mechanics.

integral

$$I(\psi) = \int \psi(t)^*(i\hbar\partial_t - H)\psi(t)dt = \int \left(i\hbar\psi(t)^*\dot{\psi}(t) - \psi(t)^*H\psi(t) \right) dt \quad (5)$$

is stationary iff ψ satisfies the time-dependent Schrödinger equation $i\hbar\dot{\psi}(t) = H\psi(t)$. Suppose now that a family of pure states ϕ_z (depending smoothly on a collection z of parameters) is believed to approximate the class of states realized in nature we may make the coarse-graining ansatz

$$\psi(t) = \phi_{z(t)}$$

and determine the time-dependent parameters $z(t)$ by finding the differential equation for the stationary points of $I(\phi_z)$ varied over all smooth functions $z(t)$. This variational principle was first used by DIRAC [21] and FRENKEL [26], and found numerous applications; a geometric treatment is given in KRAMER & SARACENO [41].

Decoherence (see, e.g., SCHLOSSHAUER [64, 65]) is a typical phenomenon arising in coarse-grained models of detailed quantum systems involving a large environment. It shows that in a suitable reduced description, the density operators soon get very close to diagonal, recovering after a very short decoherence time a Koopman picture of classical mechanics. Thus decoherence provides in principle (though only few people think of it in these terms) a reduction of the quantum physics of an open system to a highly nonlinear classical stochastic process.

For how coarse-graining is done in more general situations given a fundamental quantum field theoretic description, see, e.g., BALIAN [4], GRABERT [28], RAU & MÜLLER [61]. In general, once the choice of the resolution of modeling is fixed, this fixes the amount of approximation tolerable in the ansatz, and hence the necessary list of extensive quantities. What is necessary is not always easy to see but can often be inferred from the practical success of the resulting coarse-grained model.

4.3 Chaos, randomness, and quantum measurement

Many coarse-grained models are chaotic. In general, deterministic chaos, as present in classical mechanics, results in empirical randomness. For example, the Navier–Stokes equations, used in practice to model realistic fluid flow, are well-known to be chaotic. They exhibit stochastic features that make up the phenomenon of turbulence.

In the thermal interpretation of quantum physics, empirical randomness is also taken to be an emergent feature of deterministic chaos implicit in the deterministic dynamics of the Ehrenfest picture discussed in Part II [51]. Since the Ehrenfest dynamics is linear, it seems to be strange to consider it chaotic. However, the chaotic nature appears once one restricts attention to the macroscopically relevant q-expectations, where the influence of the ignored beables is felt as a stochastic contribution to the effective coarse-grained dynamics of the relevant q-expectations.

To explain the randomness inherent in the measurement of quantum observables in a qualitative way, the chaoticity of coarse-grained approximations to equations of motion seems to

be sufficient. The latter shows how the deterministic dynamics of the density operator gives rise to stochastic features at the coarse-grained level. The quantitative derivation of the stochastic properties is therefore reduced to a problem of quantum statistical mechanics.

The dynamics we actually observe is the quantum dynamics of a more complex system, coarse-grained to a dynamics of these few degrees of freedom – at increasing level of coarse-graining described by Kadanoff–Baym equations, Boltzmann-type kinetic equations, and hydrodynamic equations such as the Navier–Stokes equations. These coarse-grained systems generally behave like classical dynamical systems with regimes of highly chaotic motion.

In general, deterministic chaos manifests itself once one uses a coarse-grained, locally finite-dimensional parameterization of the quantum states. This leads to an approximation where, except in exactly solvable systems, the parameters characterizing the state of the universe (or a selected part of it) change dynamically in a chaotic fashion.

ZHANG & FENG [79] used the Dirac–Frenkel variational principle introduced in Subsection 4.2, restricted to group coherent states, to get a coarse-grained system of ordinary differential equations approximating the dynamics of the q -expectations of macroscopic operators of certain multiparticle quantum systems. At high resolution, this deterministic dynamics is highly chaotic. While this study makes quite special assumptions, it illustrates how although the basic dynamics in quantum physics is linear, chaotic motion results once attention is restricted to a tractable approximation. This chaoticity is indeed a general feature of coarse-graining approximation schemes for the dynamics of q -expectations or the associated reduced density functions. (For a discussion of quantum chaos from a completely different perspective see PERES [56, p.353ff] and the survey by HAAKE [30].)

According to the thermal interpretation, quantum physics is the basic framework for the description of objective reality (including everything reproducible studied in experimental physics), from the smallest to the largest scales. In particular, quantum physics must give an account of whatever happens in an experiment, when both the equipment and the systems under study are modeled on the quantum level. In experiments probing the foundations of quantum physics, one customarily observes a small number of field and correlation degrees of freedom (often simplified in a few particle setting) by means of macroscopic equipment. To model the observation of such a tiny quantum system by a macroscopic detector one must simply extend the coarse-grained description of the detector by adding a few additional quantum degrees of freedom for the measured system, together with the appropriate interactions. The metastability needed for a reliable quantum detector (e.g., in a bubble chamber) together with chaoticity then naturally leads to a random behavior of the individual detection events.

In terms of the thermal interpretation, the **measurement problem** – how to show that an experimentally assumed relation between measured system and detector results is actually consistent with the quantum dynamics – becomes a precise problem in quantum statistical mechanics.⁷ Of course, details must be derived in a mathematical manner from the

⁷On the other hand, a somewhat ill-posed, vexing measurement problem arises when one insists on the rigid, far too idealized framework in which quantum physics was developed historically and in which it is

theoretical assumptions inherent in the formal core.

A number of recent papers by ALLAHVERDYAN, BALIAN & NIEUWENHUIZEN (in the following short AB&N), reviewed in NEUMAIER [49], addressed this issue. Here we only discuss AB&N's paper [2], which carefully analyzed the assumptions regarding the statistical mechanics used that actually go into the analysis in their long, detailed paper [1] of a slightly idealized but on the whole realistic measurement process formulated completely in terms of quantum dynamics.

To avoid circularity in their arguments, AB&N introduce the name **q-expectation value** for $\langle A \rangle := \text{Tr} \rho A$ considered as a formal construct rather than a statistical entity, and similarly (as we do in Footnote ¹) q-variance and other q-notions, to be distinguished from their classical statistical meaning. This allows them to use the formalism of statistical mechanics without any reference to prior statistical notions. The statistical implications are instead derived from the analysis within this formal framework (together with explicitly specified interpretation rules), resulting in a derivation of Born's rule and the time scales in which the implied correlations of microscopic state and measurement results are dynamically realized, based on a unitary dynamics of the full quantum system involving the microscopic system, the measurement device, and a heat bath modeling the environment.

Most important for the interpretation in [2] is AB&N's "interpretative principle 1":

ABN principle: If the q-variance of a macroscopic observable is negligible in relative size its q-expectation value is identified with the value of the corresponding macroscopic physical variable, even for an individual system.

This is just a special case of the basic uncertainty principle central to the thermal interpretation of quantum physics!

4.4 Gibbs states

The detailed state of a quantum system can be found with a good approximation only for fairly stationary sources of very small objects, of which sufficiently many can be prepared in essentially the same quantum state. In this case, one can calculate sufficiently many expectations by averaging over the results of multiple experiments on these objects, and use these to determine the state via some version of quantum state tomography [78]. Except in very simple situations, the result is a mixed state described by a density operator. Mixed states are necessary also to properly discuss properties of subsystems (see Part II [51]) and for the realistic modeling of dissipative quantum systems by equations of Lindblad type (LINDBLAD [44]). Even for the multi-photon states used to experimentally check the foundations of quantum physics, quantum opticians use density operators and not wave functions, since the latter do not provide the efficiency information required to rule out loopholes.

Although only a coarse-grained description of a macroscopic system can be explicitly known, this does not mean that the detailed state does not exist. *The existence of an exact state for*

typically introduced in textbooks.

large objects has always been taken as a metaphysical but unquestioned assumption. Even in classical mechanics, it was always impossible to know the exact state of the solar system with sun, planets, asteroids, and comets treated as rigid bodies). But before the advent of quantum mechanics shattered the classical point of view, its existence was never questioned.

Motivated by the above considerations, the thermal interpretation takes as its ontological basis the density operators, the states occurring in the statistical mechanics, rather than the pure states figuring in traditional quantum physics built on top of the concept of a wave function.

In the thermal interpretation, all realistic⁸ states are described (as in quantum statistical mechanics) by **Gibbs states**, i.e., density operators of the form

$$\rho := e^{-S/\bar{k}}, \quad (6)$$

where \bar{k} the Boltzmann constant and S is a self-adjoint Hermitian quantity called the **entropy** of the system in the given state. (The traditional entropy is the uncertain value $\langle S \rangle$ of the present quantity S .) Note that a unitary transform $\rho' = U\rho U^*$ of a Gibbs state by a unitary operator U is again a Gibbs state; indeed, the entropy of the transformed state is simply $S' = USU^*$. This shows that the notion of a Gibbs state is dynamically well-behaved; the von Neumann dynamics ensures that we get a consistent evolution of Gibbs states.

On the level of Gibbs states, the notion of superposition becomes irrelevant; one cannot superimpose two Gibbs states. Pure states, where superpositions are relevant, appear only in a limit where the entropy operator has one dominant eigenvalue and then a large spectral gap. For example, as we have seen in Part I [50, Subsection 2.2] of this series of papers, this is approximately the case for equilibrium systems where the Hamiltonian has a nondegenerate ground state and the temperature is low enough. For this one needs a sufficiently tiny system. A system containing a screen or a counter is already far too large.

The simplest and perhaps most important case of a Gibbs state is that of an **equilibrium state** of a pure substance, defined by the formula

$$S = (H + PV - \mu N)/T,$$

where H is the Hamiltonian, V is the system volume, N a nonrelativistic number operator, and **temperature** T , **pressure** P , and **chemical potential** μ are parameters. This represents equilibrium states in the form of density operators corresponding to grand canonical ensembles, $\rho = e^{-\beta(H+PV-\mu N)}$, where $\beta = 1/\bar{k}T$.

A derivation of equilibrium thermodynamics in terms of grand canonical ensembles in the spirit of the thermal interpretation is given in Chapter 10 of NEUMAIER & WESTRA [53].

⁸This excludes more idealized states, for example pure states. All states, including the idealized ones, are obtainable as limits of Gibbs states. This is because the positive definite density operators are dense in the set of all density operators, and every positive definite density operator is a Gibbs state. Indeed, being trace class and Hermitian, a density operator is self-adjoint, and positive definiteness implies the existence of the self-adjoint entropy operator $S = -\bar{k} \log \rho$, showing that (6) holds. In particular, it is experimentally impossible to distinguish between a pure state and Gibbs states sufficiently close to the pure state.

In this development there is no mention of size. The latter matters only when one wants to conclude exact thermodynamic results, since then the thermodynamic limit (infinite volume limit) has to be taken to reduce the uncertainty to zero.

Temperature T , pressure P , and chemical potential μ have no simple description in terms of microscopic variables. They figure only as a parameter in the expression for the grand canonical phase space density $\rho = e^{-(H+PV-\mu N)/kT}$ of the state. But T and P are computable from ρ via the thermodynamic formalism of statistical mechanics, and hence are beables.

The definition (M) from Subsection 2.1 of what it means to measure something therefore applies. More generally, it applies (see NEUMAIER & WESTRA [53]) to arbitrary macroscopic thermal systems in equilibrium, whose state is characterized by a collection of finitely many extensive and intensive thermodynamic variables related by the standard thermodynamic relations expressed in terms of an equation of state for the materials making up the thermal system.

In particular, the measurement of temperature and pressure of, say, a single brick of iron in equilibrium is a perfectly sensible special case of our definition (M) of what it means to measure something. On the other hand, according to the traditional interpretations, they are not even "observables" – although they are observable in any meaningful sense of the word!

A realistic system is never exactly in equilibrium, but if it is sufficiently close to equilibrium, the entropy S is well approximated by its equilibrium expression $(H + PV - \mu N)/T$. The residual term $H + PV - \mu N - ST$, which vanishes at equilibrium, contains the detailed information thrown away in the equilibrium approximation.

4.5 Nonequilibrium statistical mechanics

Unlike in traditional classical or quantum statistical mechanics, the density operator (6) is regarded in the thermal interpretation as the complete, exact description of the state, not a coarse-grained one. However, one obtains a coarse-grained reduced description by replacing the exact S with a suitable approximate S given by a more tractable parameterized expression.

In most coarse-grained models used in statistical mechanics, the form assumed for the entropy operator S is a linear combination of **relevant quantities** whose q-expectations define the **extensive variables** of the description. The corresponding coefficients are parameters characterizing the particular state of the reduced system; they are referred to as the **intensive variables** of the description. Extensive variables scale linearly with the size of the system (which might be mass, or volume, or another additive parameter), while intensive variables are invariant under a change of system size. We do *not* use the alternative convention to call extensive any variable that scales linearly with the system size, and intensive any variable that is invariant under a change of system size.

If the relevant quantities depend on continuous variables, which is the case in nonequilibrium

situations, the extensive and intensive variables become fields depending on the continuum variables used to label the subsystems. For extensive variables, the integral of their field quantities over the label space gives the bulk value of the extensive quantity; thus the fields themselves have a natural interpretation as a **density**. For intensive variables, an interpretation as a density is physically meaningless; instead, they have a natural interpretation as **field strengths**, sources for **thermodynamic forces** given by their gradients.

In statistical mechanics, we distinguish four nested levels of thermal descriptions, depending on whether the system is considered to be in global, local, microlocal, or quantum equilibrium. The highest and computationally simplest level, **global equilibrium**, is concerned with macroscopic situations characterized by finitely many space- and time-independent variables.

The next level, **local equilibrium**, treats macroscopic situations in a continuum mechanical description, leading, e.g., to the Navier–Stokes equations of fluid mechanics. Here the equilibrium subsystems are labeled by the space coordinates. Therefore the relevant variables are finitely many space- and time-dependent fields. The entropy operator S becomes time-dependent as is represented as a spatial integral

$$S(t) := \int s(t, x) dx$$

with a spatial entropy density $s(t, x)$. For a pure monatomic substance, the latter is in the nonrelativistic case of the form

$$s(t, x) = \left(\varepsilon(t, x) + p(t, x) - \mu(t, x)\rho(t, x) \right) / T(t, x),$$

where $\varepsilon(t, x)$ and $\rho(t, x)$ are the internal energy density and the mass density operators of a quantum field theory whose expectations give extensive densities, and $T(t, x)$, $p(t, x)$, and $\mu(t, x)$ are intensive coefficient fields defining the local temperature, pressure, and chemical potential. (In the relativistic case, similar but more involved expressions are used, and the identification of temperature and pressure is convention-dependent.)

The next deeper level, **microlocal**⁹ **equilibrium**, treats mesoscopic situations in a kinetic description, where the equilibrium subsystems are labeled by phase space coordinates. This leads, e.g., to the Boltzmann equation or the Kadanoff–Baym equations. The relevant variables are now finitely many fields depending on time, position, and momentum; cf. BALIAN [4] and RAU & MÜLLER [61]. Now the entropy operator S is represented (in the nonrelativistic case) as a phase space integral

$$S(t) := \int s(t, x, p) dx dp$$

with a phase space entropy density $s(t, x, p)$ linearly expressed in terms of Wigner-transformed operators of a quantum field theory whose expectations give extensive phase space densities.

⁹The term microlocal for a phase space dependent analysis is taken from the literature on partial differential equations; see, e.g., MARTINEZ [48].

The bottom level is the microscopic regime, where we must consider **quantum equilibrium**. This no longer fits a thermodynamic framework but must be described in terms of quantum dynamical semigroups and dynamical equations of Lindblad type (LINDBLAD [44]).

Each description level may be considered as a special case of each more detailed description level. For example, global equilibrium is a special case of local equilibrium; the extensive variables in the single-phase global equilibrium case have constant densities.

Table 1: Typical conjugate pairs of thermal variables and their contribution to the Euler equation. The signs are fixed by tradition. (In the gravitational term, m is the vector with components m_j , the mass of a particle of kind j , g the acceleration of gravity, and h the height.)

extensive X_j	intensive α_j	contribution $\alpha_j X_j$
entropy S	temperature T	thermal, TS
particle number N_j	chemical potential μ_j	chemical, $\mu_j N_j$
conformation tensor C	relaxation force R	conformational $\sum R_{jk} C^{jk}$
strain ε^{jk}	stress σ_{jk}	elastic, $\sum \sigma_{jk} \varepsilon^{jk}$
volume V	pressure $-P$	mechanical, $-PV$
surface A_S	surface tension γ	mechanical, γA_S
length L	tension J	mechanical, JL
displacement q	force $-F$	mechanical, $-F \cdot q$
momentum p	velocity v	kinetic, $v \cdot p$
angular momentum J	angular velocity Ω	rotational, $\Omega \cdot J$
charge Q	electric potential Φ	electrical, ΦQ
polarization P	electric field strength E	electrical, $E \cdot P$
magnetization M	magnetic field strength B	magnetical, $B \cdot M$
electromagnetic field F	electromagnetic field strength $-F^s$	electromagnetic, $-\sum F_{\mu\nu}^s F^{\mu\nu}$
mass $M = m \cdot N$	gravitational potential gh	gravitational, ghM
energy-momentum U	metric g	gravitational, $\sum g_{\mu\nu} U^{\mu\nu}$

In phenomenological approaches to nonequilibrium thermodynamics, the entropy operator is written as a linear combination

$$S = (H - \sum_j \alpha_j X_j)/T,$$

of relevant extensive quantities X_j , when space is not resolved, and a corresponding density form when space is resolved to local equilibrium. (In microlocal equilibrium, temperature T is no longer well-defined, and the linear combination is written differently.) In each case, the relevant quantities are precisely those variables that are observed to make a difference in modeling the phenomenon of interest. Table 1 gives typical extensive variables (S and X_j), their intensive conjugate variables (T and α_j), and their contribution (TS and $\alpha_j X_j$) to the Euler equation

$$H = TS + \sum_j \alpha_j X_j \quad (7)$$

resulting from the definition of the entropy. Some of the extensive variables and their intensive conjugates are vectors or (in elasticity theory, the theory of complex fluids, and in the relativistic case) tensors; cf. BALIAN [3] for the electromagnetic field and BERIS & EDWARDS [7], ÖTTINGER [55] for complex fluids.

In general, which quantities need to be considered depends on the resolution with which the system is to be modeled – the higher the resolution, the larger the family of extensive quantities. Whether we describe bulk matter, surface effects, impurities, fatigue, decay, chemical reactions, or transition states, – the general setting remains the same since it is a universal approximation scheme, while the number of degrees of freedom increases with increasingly detailed models.

In practice, relevant quantities and corresponding states are assigned to real life situations by well-informed judgment concerning the behavior of the equipment used. The validity of the assignment is experimentally tested by comparing experimental results with the chosen mathematical model. The model defines the meaning of the concepts: the theory defines what an object is.

For example, a substance is regarded as an ideal gas if it behaves to a satisfactory degree like the mathematical model of an ideal gas with certain values of temperature, pressure and volume. Similarly, a solid is regarded as a crystal if it behaves to a satisfactory degree like the mathematical model of a crystal for suitable numerical values of the model parameters.

In general, as put by the author of one of the most influential textbooks of thermodynamics: *”Operationally, a system is in an equilibrium state if its properties are consistently described by thermodynamic theory.”* (CALLEN [17, p.15]) At first sight, this sounds like a circular definition. But this is not the case since the formal meaning of *”consistently described by thermodynamic theory”* is already known. The operational definition simply moves it from the domain of theory to the domain of reality by defining when a system deserves the designation *”is in an equilibrium state”*. In particular, this definition allows one to determine experimentally whether or not a system is in equilibrium.

In general, we know or assume on the basis of past experience, claims of manufacturers, etc., that certain materials or machines reliably produce states that, to a satisfactory degree for the purpose of the experiment or application, depend only on variables that are accounted for in our theory and that are, to a satisfactory degree, either fixed or controllable. The nominal state of a system can be checked and, if necessary, corrected by **calibration**, using appropriate measurements that reveal the parameters characterizing the state.

All this is completely independent of a stochastic setting (although the name ‘statistical mechanics’ would suggest something different), and one gets consistent results that compare well with experiment (if the requirements for the validity of the classical treatment of certain degrees of freedom are met). Everything can be proved (at least at the level of typical theoretical physics derivations), and it has all the beauty and usefulness one might want to have.

4.6 Conservative mixed quantum-classical dynamics

The Koopman representation makes classical systems look quantum. It is also possible to make quantum systems look classical. The resulting quantum-classical dynamics has important applications.

Since the differences between classical mechanics and quantum mechanics disappear in the Ehrenfest picture in favor of the common structure of a classical Hamiltonian dynamics, we can use this framework to mix classical mechanics and quantum mechanics. The resulting **quantum-classical dynamics** is described in many places, e.g., in PERES & TERNO [58], KAPRAL & CICCOTTI [39], PREZDHO & KISIL [60], PREZDHO [59], BREUER & PETRUCCIONE [15]. The derivation of quantum-classical dynamics from pure quantum dynamics in these papers follows (under well-understood conditions) from the principles of statistical mechanics of q-expectations as embodied in the thermal interpretation, and does not depend on any measurement issues. Thus it remains valid without any change.

There are many systems of practical interest which are treated in a hybrid quantum-classical fashion, where the most slowly varying degrees of freedom are treated classically, whereas the most rapidly oscillating ones are treated in a quantum way. It is important to have an interpretation in which this can be consistently interpreted. Any hybrid theory must be interpreted in terms of concepts that have identical form in classical and in quantum mechanics; otherwise there are inevitable conflicts. This is, however, impossible in the traditional statistical interpretation; there are several theorems in the literature documenting this [14, 63]. On the other hand, the thermal interpretation can cope successfully with this challenge. It is a theory which contains the classical and the quantum case as two special cases of the same conceptual framework. In this framework one can therefore discuss things consistently that lead to puzzles if interpreted either on a pure classical or on a pure quantum basis, or in some ill-defined in-between limbo.

The basic equations for a large class of quantum-classical models are, in the Schrödinger picture, the **Liouville equation**

$$i\hbar\dot{\rho} = [H(p, q), \rho] \quad (8)$$

and the **Hamilton equations**

$$\dot{q} = \text{Tr} \rho \frac{\partial}{\partial p} H(p, q), \quad \dot{p} = -\text{Tr} \rho \frac{\partial}{\partial q} H(p, q). \quad (9)$$

Here $q = q(t), p = p(t)$ are classical, time-dependent variables, not quantum operators, $H(p, q)$ is, for any fixed p, q , a linear operator on some Euclidean space \mathbb{H} of smooth wave functions, and $\rho = \rho(t)$ is a time-dependent density operator on \mathbb{H} . The sufficiently nice functions of q-expectations

$$\langle A(p, q) \rangle = \text{Tr} \rho A(p, q) \quad (10)$$

where A is a (p, q) -dependent operator on a nuclear space, are classical quantities forming a commutative algebra. In terms of q-expectations, we have

$$\dot{A} = \langle H \angle A \rangle,$$

and in particular,

$$\dot{q} = \left\langle \frac{\partial}{\partial p} H(p, q) \right\rangle, \quad \dot{p} = - \left\langle \frac{\partial}{\partial q} H(p, q) \right\rangle,$$

This looks like the original form of the Ehrenfest equations (Part II [51, eq. (9)]), except that on the left hand side we have classical variables and no expectations. The expected energy $\langle H(p, q) \rangle$ is conserved.

The quantum-classical dynamics preserves the rank of the density ρ . In particular, if ρ has the rank 1 form

$$\rho = \psi\psi^* \tag{11}$$

at some time, it has at any time the form (11) with time-dependent ψ . The fact that ρ has trace 1 translates into the statement that the state vector ψ is normalized to $\psi^*\psi = 1$. As discussed in detail in Part I [50, Subsection 2.3], the Liouville equation (8) holds iff the state vector ψ , determined by (11) up to a phase, satisfies – for a suitable choice of the phases – the Schrödinger equation

$$i\hbar\dot{\psi} = H(p, q)\psi.$$

In terms of the state vector, q-expectations now take the familiar form

$$\langle A(p, q) \rangle = \psi^* A(p, q)\psi.$$

In general, q-expectations in the quantum-classical dynamics are to be interpreted as objects characterizing a single quantum system, in the sense of the thermal interpretation, and not as the result of averaging over many realizations. The quantum-classical dynamics is commonly discussed in the Schrödinger picture, but it is independent of the picture used. The equivalent Heisenberg dynamics is

$$\frac{d}{dt}A = \frac{\partial A}{\partial q} \left\langle \frac{\partial H}{\partial p} \right\rangle - \frac{\partial A}{\partial p} \left\langle \frac{\partial H}{\partial q} \right\rangle + \frac{i}{\hbar}[H, A]$$

where now $\langle \cdot \rangle$ is the fixed Heisenberg state. From this, one can immediately see that everything depends only on q-expectations by taking expectations in this equation,

$$\frac{d}{dt}\langle A \rangle = \left\langle \frac{\partial A}{\partial q} \right\rangle \left\langle \frac{\partial H}{\partial p} \right\rangle - \left\langle \frac{\partial A}{\partial p} \right\rangle \left\langle \frac{\partial H}{\partial q} \right\rangle + \left\langle \frac{i}{\hbar}[H, A] \right\rangle. \tag{12}$$

This is now a fully deterministic equation for q-expectations of the mixed quantum-classical model, considered in the Ehrenfest picture. This is now the most natural picture, since we still get a Hamiltonian description of the form

$$\frac{d}{dt}\langle A \rangle = \langle H \rangle \angle \langle A \rangle. \tag{13}$$

But now the Lie algebra is the direct product of the Lie algebra of the classical subsystem and the Lie algebra of the quantum subsystem. This results in a nonlinear dependence on

expectations. Such nonlinearities are common for reduced descriptions obtained by coarse-graining (cf. Subsection 4.4 below), both from a pure quantum theory or from a classical stochastic theory (in the Koopman representation discussed below in Subsection 4.1. Since quantum-classical systems (at least as they appear in the literature) are also coarse-grained descriptions, there is nothing surprising in that the same phenomenon occurs.

In the Schrödinger picture and the Heisenberg picture, the description of a quantum-classical system looks different from that in the purely classical and purely quantum case.

New in quantum-classical systems – compared to pure quantum dynamics – is that in the Heisenberg picture, the Heisenberg state occurs explicitly in the differential equation for the dynamics. But it does not take part in the dynamics, as it should be in any good Heisenberg picture. The state dependence of the dynamics is not a problem for practical applications since the Heisenberg state is fixed anyway by the experimental setting.

This makes an important difference in the interpretation of the theory. In contrast to the pure quantum case, there is now a difference between averaging results of two experiments ρ_1, ρ_2 and the results of a single experiment ρ given by $(\rho_1 + \rho_2)/2$. That, in ordinary quantum theory, the two are indistinguishable in their statistical properties is a coincidental consequence of the linearity of the Schrödinger equation, and the resulting state independence of the Heisenberg equation; it does no longer hold in effective quantum theories where nonlinearities appear due to a reduced description. Since quantum-classical systems (at least as they appear in the literature) also are reduced descriptions, there is nothing surprising in that the same phenomenon occurs.

Because the dynamics depends on the Heisenberg state, calculating results by splitting a density at time $t = 0$ into a mixture of pure states no longer makes sense. One gets different evolutions of the operators in different pure states, and there is no reason why their combination should at the end give the correct dynamics of the original density. (And indeed, this will usually fail.) This splitting is already artificial in pure quantum mechanics since there is no natural way to tell of which pure states a mixed state is composed of. But there the splitting happens to be valid and useful as a calculational tool since the dynamics in the Heisenberg picture is state independent.

In the quantum-classical case, not even this is possible, so the quantum-classical equations have no sensible interpretation in terms of mixing pure cases into an ensemble. Thus the quantum-classical setting cannot be consistently interpreted in the traditional interpretations, where q-expectations have only a statistical meaning. But in the thermal interpretation, this is not a problem since densities are irreducible objects describing a single quantum system, not stochastic entities that make sense only under repetition. Thus in the thermal interpretation, the quantum-classical setting is very natural.

It is in principle conceivable (though not desirable from the point of view of simplicity) that the most fundamental description of nature is truly quantum-classical and not purely quantum. In the absence of an interpretation with a consistent quantum-classical setting, this would have been unacceptable, but apart from elegance, there are no longer fundamental reasons that would forbid it.

4.7 Important examples of quantum-classical dynamics

There are many examples of quantum-classical dynamics of practical importance.

Probably the most important quantum-classical system is a version of the Born–Oppenheimer approximation of molecules, widely used in quantum chemistry. Here the nuclei are described in terms of classical phase space variables, while the electrons are described quantum mechanically by means of a state vector ψ in a Hilbert space of antisymmetrized electron wave functions.

A spinning relativistic electron, while having no purely classical description, can be modeled quantum-classically by classical phase space variables p, q and a quantum 4-component spin with Hamiltonian

$$H(p, q) = \alpha \cdot p + \beta m + eV(q) \quad (14)$$

is a 4×4 matrix parameterized by classical 3-vectors $p = p(t)$ and $q = q(t)$, $\rho = \rho(t)$ is a positive semidefinite 4×4 matrix of trace 1, and the trace in equation (10) is just the trace of a 4×4 matrix. One gets the equations (8) and (9) from Dirac’s equation and Ehrenfest’s theorem by an approximation involving coherent states for position and momentum. This is just a toy example; more useful field theoretic quantum-classical versions (see, e.g., GÉRARD et al. [27]) lead to well-known Vlasov equations for (p, q) -dependent 4×4 densities, describing a fluid of independent classical electrons of the form (14).

Other examples include the Schrödinger–Poisson equations in semiconductor modeling and the quantum Boltzmann equation with spin represented by 4×4 (or in the nonrelativistic case 2×2) matrices parameterized by classical phase space variables. (On the other hand, the quantum-Boltzmann equation for spin zero is already a purely classical equation, since all its dynamical variables are mutually commuting.)

With even more realism, one needs to add to quantum-classical descriptions (cf. Subsection 4.6) a dissipative collision term accounting for interactions, and (14) is no longer adequate but needs additional stochastic terms.

5 The relation to traditional interpretations

Über die physikalische Interpretation der Formeln sind die Meinungen geteilt.

Max Born, 1926 [11, p.803]

Das Einzelsystem trägt wirklich die Fähigkeit in sich, einem bestimmten Meßvorgang gegenüber in verschiedener Weise zu reagieren, d.h. verschiedene Meßwerte für ein und dieselbe Zustandsgröße zu liefern: welchen, hängt vom Zufall ab, oder besser wohl von den Phasenbeziehungen zwischen dem System und dem Meßinstrument.

Erwin Schrödinger, 1929 [20, Vorwort]

I reject the basic idea of contemporary statistical quantum theory, insofar as I do not believe that this fundamental concept will provide a useful basis for the whole of physics.

Albert Einstein, 1949 [25]

It is usually believed, that the current orthodox theory actually accounts for the 'nice linear traces' observed in the Wilson chamber etc. I think this is a mistake, it does not.

Erwin Schrödinger, 1958 [66, p.163]

Personally I still have this prejudice against indeterminacy in basic physics.

Paul Dirac, 1972 [22, p.7]

When it comes to specifying exact details, one discovers that we cannot rigorously define what quantum mechanical amplitudes are, what it means when it is claimed that 'the universe will collapse with such-and-such probability', what and where the observers are, what they are made of, and so on. Yet such questions are of extreme importance if one wants to check a theory for its self-consistency, by studying unitarity, causality, etc.

Gerard 't Hooft, 1999 [73, p.95]

My own conclusion (not universally shared) is that today there is no interpretation of quantum mechanics that does not have serious flaws.

Steven Weinberg, 2013 [74, p.95]

From its very beginning in 1926, what turned out to be the formal core of quantum mechanics had conflicting interpretations – initially the deterministic view of Schrödinger and the statistical view of Born. In 1929, Schrödinger conceded the need for a statistical interpretation. But the details remained controversial. Today, after almost 100 years of interpretation quarrels, the matter is still not settled. As the above quotes show, many of the founders of quantum mechanics were never satisfied with the interpretation of quantum mechanics, and even some of today's Nobel prize winners spend significant effort on the interpretation issue.¹⁰

A multitude of interpretations of quantum mechanics exist; most of them in several variants. We distinguish the following types:

(I) **Individual interpretations** (such as certain variants of the Copenhagen interpretation), where the state of a system is determined by an individual realization of the system and contains the information about everything that can possibly be said about it.

(S) **Statistical interpretations** (such as the minimal interpretation), where the state of a system says (except in special cases) nothing about a single system but is only about statistical predictions of actual measurements on an ensemble of similarly prepared systems.

¹⁰But apparently they did so only after their retirement: While paid they researched more important issues and kept – like most quantum physicists – the foundational issues on the back burner.

(K) **Knowledge interpretations**, where a state says nothing objective about the systems modeled, but is only about the subjective knowledge of these system.

(O) **Other interpretations**, where a state consists (as in Bohmian mechanics) of more than a state vector or density operator, or is (as in many worlds interpretations) by their conception about more than actual events recorded in actual experiments.

In the mainstream interpretations of the types (I) and (S), the result of a single measurement is – in contrast to classical mechanics – not even theoretically determined before the measurement is done.¹¹

As we shall see in this section, the mainstream interpretations may be regarded as partial versions of the thermal interpretation. In particular, certain puzzling features of both the Copenhagen interpretation and the statistical interpretation get their explanation through the thermal interpretation of quantum field theory. We shall see that these peculiar features get their natural justification in the realm for which they were created – the statistics of few particle scattering events.

Interpretations of the types (K) and (O) have little in common with the thermal interpretation and are not further discussed.

5.1 The statistical mechanics of definite, discrete events

Generally in physics, invariance and the resulting reproducibility determine what counts as an objective property of what: In 3-dimensional vision, observed length is not a property of an observed object by itself but a property of the object and the distance from the observer. Extrapolation to zero distance defines an invariant objective length. In relativity, length is no longer a property of an observed object and the distance from the observer but a property of the object and the distance from and relative speed to the observer. Extrapolation to zero distance and zero velocity defines the invariant objective length.

Science is about reproducible aspects of our world, and hence not all permanent records but only reproducible results count as measurement results. This is the main difference between the thermal interpretation and traditional interpretations of quantum mechanics.

As a consequence, a measurement of a Hermitian quantity A gives an uncertain value approximating the q-expectation $\langle A \rangle$ rather than (as tradition wanted to have it) an exact eigenvalue of A . This difference is most conspicuous in the interpretation of single discrete events. Since most single microscopic observations are not reproducible they have no scientific value in themselves, and do not constitute measurement results.¹² Scientific value is, however, in ensembles of such observations, which result in approximate measurements of q-probabilities and q-expectations.

¹¹However, expositions of both views generally prefer to remain vague or even silent about this.

¹²The same holds in classical stochastic models. If die casting is part of a stochastic system description, the single die cast tells nothing about the state of the model and hence is of no value for the scientific study of the model.

In the thermal interpretation, the traditional difficulty to show that there is always a unique outcome is trivially solved since by definition, the outcome of reading a macroscopic quantity is its expectation value, with negligible uncertainty. Instead we now have a new difficulty absent in traditional interpretations: An explanation is required why, although fed with a stationary interaction, certain detectors record random individual events!

For example, why does a low intensity beam of light produce in a photodetector a discrete signal? The uncertain observed value is the q -expectation of a photocurrent, which a priori has a continuum of possible values. But observed are two clearly different regimes that allow one to clearly distinguish between the occurrence and the nonoccurrence of a detection event. In the thermal interpretation, we do not consider the single detector event as a property of the observed beam ("a particle arrived through the beam"), since only the statistics of an ensemble of detector events (e.g., a Poisson distribution of the number of events in some large time interval) is reproducible and hence constitutes an objective property of the beam. But why these discrete events can be clearly distinguished at all needs an explanation.

Section 6.6 of the book on open quantum systems by BREUER & PETRUCCIONE [16, pp.348–350] (in the following short B&P) addresses this issue. The dynamics of a large quantum system, consisting of an observed system and a detector observing it, is treated there as a classical dynamical system for the density operator with stochastic initial conditions, and reduced by appropriate coarse-graining to a classical stochastic equation for the coarse-grained stochastic density operator. The derivation is done using standard assumptions from classical statistical mechanics only, in the same way as one would proceed in statistical mechanics for any other classical dynamical system.

The detector must include enough of the environment to produce irreversible results (and hence determines what is read out). B&P model the latter by assuming separated time scales and the validity of the Markov approximation - which hold only if the detector is big enough to be dissipative. (The latter is typically achieved by including in the detector a heat bath consisting of an infinite number of harmonic oscillators.) Since B&P make these assumptions without deriving them, their analysis holds for general dissipative detectors. But – as always in statistical mechanics – one must check for any concrete application that these assumptions are plausible.

In sufficiently idealized settings, these assumptions can actually be proved rigorously, but this is beyond the scope of the treatment by B&P. Rigorous results (without the discussion of selective measurement but probably sufficient to establish the assumptions used by B&P) were first derived by Davies 1974 and later papers with the same title. See also the detailed survey by SPOHN [68].

The stochastic equations discussed by B&P preserves the rank of the density operator, and hence can be applied to pure states, where the dynamic reduces in general to that of a piecewise deterministic stochastic process (PDP), a diffusion process, or a combination of both. The piecewise deterministic part accounts for the statistics of **discrete events**.

In the cases treated by B&P in Chapter 6 (usually for the pure case only), the PDP corresponds to photodetection, which measures the particle number operator (with a discrete spectrum); the diffusion processes correspond to homodyne or heterodyne detection, which measure quadratures (with a continuous spectrum). B&P obtain the latter from the PDP

by a limiting process in the spirit of the traditional approach treating a continuous spectrum as a limit of a discrete spectrum.

But although the pointer reading is a position measurement of the pointer, what is measured about the particle is not its position but the variable correlated with the pointer reading – the photon number or the quadrature. Particle position is as indeterminate as before. Indeed, investigation of the PDP process shows that the collapsed states created by the PDP are approximate eigenstates of the number operator or the quadrature. Thus the PDP can be interpreted in Copenhagen terms as constituting the repeated measurement of particle number or quadrature.

For photodetection, one gets at the end a PDP for the reduced state vector, only using classical probabilities in the whole derivation. But after everything has been done, the PDP may be interpreted in terms of quantum jumps, without having postulated any irreducible "collapse" as in the Copenhagen interpretation (cf. Subsection 5.3). This suggests that, in general, that collapse in a single observed system – in the modern POVM version of the von Neumann postulates for quantum dynamics – is derivable from the unitary dynamics of a bigger system under the standard assumptions that go into the traditional derivations in classical statistical mechanics.

The arguments show that to go from unitarity to irreversible discrete events in Hamiltonian quantum mechanics one does not need to assume more than to go from reversibility to irreversibility in Hamiltonian classical mechanics – namely a suitable form of the Markov approximation. Statistical assumptions are not needed to make pointers acquire a well-defined position or to create photocurrents – the standard dissipation arguments are enough. This gives stochastic equations for definite macroscopic outcomes.

5.2 Dissipation, bistability, and Born's rule

The development of B&P is mathematical and quantitative but abstract. We now provide a qualitative explanation why the discreteness that makes its appearance in quantum mechanics is actually quite natural, explained by environment-induced randomness and the associated environment-induced dissipation. This provides a more intuitive view of how the thermal interpretation settles this foundational key issue. (For generalities about environment-induced randomness and dissipation see, e.g., the first two chapters of CALZETTA & HU [18].)

In general, dissipation in the effective, human time scales dynamics of a set of relevant variables is a frequent situation even when the fully detailed dynamics is conservative. This effective dissipation is the reason underlying the possibility of reduced, coarse-grained descriptions whenever there is a separation of time scales for slow and fast processes. Then one can coarse-grain by eliminating the fast modes and obtain a simpler limiting (effectively time-averaged) description on the slow manifold, the manifold where all slow motion happens (see, e.g., LORENZ [45], ROBERTS [62]). Whenever the slow manifold is disconnected, metastable states of the full manifold decay under uncontrollable (environment-induced) perturbations into states in one of the connected components of the slow manifold. The components thus label random events selected by environmental noise.

For example, consider bending a classical, rotationally symmetric rod using a force in the direction of the axis of the rod. If the force exceeds the threshold where the straight rod becomes metastable only, the rod will bend into a random, but definite direction. The randomness arises from the classical Hamiltonian dynamics together with the tiniest amount of noise causing a deviation from perfect symmetry. The same analysis can be made for the dynamics of a metastable inverted classical pendulum.

Similarly, perturbing in an uncontrollable way a classical bistable system arbitrarily little from the intermediate metastable state linking the two local minima of the potential leads to a tiny random move into one of the two potential wells. Even the slightest amount of dissipated energy fixes the selection of the potential well, and more dissipation forces the system after a short relaxation time to be very close to one of the two minimum position. This is the principle underlying the emergence of chemical reactions of molecules (recognizable bound states of few atoms) from a multiparticle atomic description in transition state theory (HÄNGGI et al. [31]).

Papers on optical bistability (e.g., DRUMMOND & WALLS [24], STEYN-ROSS & GARDINER [69]) show how coarse-grained bistability arises from a quantum model by projecting out irrelevant degrees of freedom. Any bistable system obtained as a reduced description from a larger unitary system behaves in the same way. Thus one expects a few-particle quantum system coupled to a macroscopic metastable instrument to behave in the same way when (as is usual) unstable stationary points are present.

Thus bistability and more general multistability, together with dissipation leads, within the accuracy of the approximations involved, to the emergence of random discrete events from deterministic dynamics. The time scale of the emergence of these discrete events is likely to be a small multiple of the decoherence time of the system; cf. SCHLOSSHAUER citeSchl.book.

The traditional introductory textbook approach to measurement is based on the concept of ideal measurements – illustrated with Stern–Gerlach experiments, low density double-slit experiments, and the like. These experiments illustrate an antiquated view of measurement, dating back to the time before 1975, when POVMs (see Subsection 2.5) were still unknown. Until then, quantum measurements used to be described solely in terms of ideal statistical measurements. These constitute a special case (or for continuum measurements a special limiting case) of POVMs where the P_k form a family of **orthogonal projectors**, i.e., linear operators satisfying

$$P_k^2 = P_k = P_k^*, \quad P_j P_k = 0 \quad \text{for } j \neq k,$$

to the eigenspaces of a self-adjoint quantity A (or the components of a vector A of commuting such quantities) with discrete spectrum given by a_1, a_2, \dots . We may call a statistical instrument for measuring A in terms of such a POVM a **Born instrument**, and the instrument is then said to perform an **ideal measurement** of A .

Ideal measurements of A have quite strong theoretical properties since under the stated assumptions, the instrument-based statistical average

$$\overline{f(A)} = p_1 f(a_1) + p_2 f(a_2) + \dots$$

agrees for all functions f defined on the spectrum of A with the model-based value $\langle f(A) \rangle$. In an ideal measurement, the relationship between the properties of the instrument and the properties of the system have a purely correlative nature, and the rule (3) defining the probabilities reduces to the discrete form (e.g., DRUMMOND & WALLS [24], STEYN-ROSS & GARDINER [69]) of **Born's rule**. On the other hand, these strong properties are bought at the price of idealization, since (unlike more general POVMs) they frequently result in effects incompatible with real measurements.

As we saw above, bistability explains the appearance of discrete binary events. Once these are given, they provide an ideal binary measurement of the statement associated with the event – giving on a single event the result 0 or 1 with a large uncertainty compared with the probability it measures. The weak law then implies, as we saw in Part II [51, Subsection 3.5], that the relative frequencies in sufficiently large samples approximate the probability for a positive event, here given by Born's rule for ideal binary measurements.

In particular, for $P_1 = \phi\phi^*$, where ϕ has norm 1, and $P_2 = 1 - P_1$, this covers the case discussed in Part II [51, Subsection 3.4], whether a quantum system in the pure state ψ responds to a test for state ϕ , and gives Born's squared probability amplitude formula $p = |\phi^*\psi|^2$ for the probability of a positive test result. When interpreted as a measure of beam intensity, this formula is identical with Malus' law from 1809 (cf. Subsection 3.5).

5.3 The Copenhagen interpretation

The concept of observation is in so far arbitrary as it depends upon which objects are included in the system to be observed. Ultimately every observation can of course be reduced to our sense perceptions. The circumstance, however, that in interpreting observations use has always to be made of theoretical notions, entails that for every particular case it is a question of convenience at what point the concept of observation involving the quantum postulate with its inherent 'irrationality' is brought in.

Niels Bohr, 1927 [8, p.580]

Um zur Beobachtung zu gelangen, muss man also irgendwo ein Teilsystem aus der Welt ausschneiden und über dieses Teilsystem eben 'Aussagen' oder 'Beobachtungen' machen. Dadurch zerstört man dort den feinen Zusammenhang der Erscheinungen und an der Stelle, wo wir den Schnitt zwischen dem zu beobachtenden System einerseits, dem Beobachter und seinen Apparaten andererseits machen, müssen wir Schwierigkeiten für unsere Anschauung erwarten. [...] Jede Beobachtung teilt in gewisser Weise die Welt ein in bekannte und unbekannte oder besser: mehr oder weniger genau bekannte Grössen.

Werner Heisenberg, 1927 [33, p.593f]

wir müssen die Welt immer in zwei Teile teilen, der eine ist das beobachtete System, der andere der Beobachter. In der ersteren können wir alle physikalischen Prozesse (prinzipiell wenigstens) beliebig genau verfolgen, in der letzteren ist dies sinnlos. Die Grenze zwischen beiden ist weitgehend willkürlich

John von Neumann, 1932 [54, p.223f]

Aus diesem Zwiespalt ergibt sich die Notwendigkeit, bei der Beschreibung atomarer Vorgänge einen Schnitt zu ziehen zwischen den Meßapparaten des Beobachters, die mit den klassischen Begriffen beschrieben werden, und dem Beobachtungsobjekt, dessen Verhalten durch eine Wellenfunktion dargestellt wird. Während nun sowohl auf der einen Seite des Schnittes, die zum Beobachter führt, wie auf der anderen, die den Gegenstand der Beobachtung enthält, alle Zusammenhänge scharf determiniert sind – hier durch die Gesetze der klassischen Physik, dort durch die Differentialgleichungen der Quantenmechanik –, äußert sich die Existenz des Schnittes doch im Auftreten statistischer Zusammenhänge. An der Stelle des Schnittes muß nämlich die Wirkung des Beobachtungsmittels auf den zu beobachtenden Gegenstand als eine teilweise unkontrollierbare Störung aufgefaßt werden. [...] Entscheidend ist hierbei insbesondere, daß die Lage des Schnittes – d.h. die Frage, welche Gegenstände mit zum Beobachtungsmittel und welche mit zum Beobachtungsobjekt gerechnet werden – für die Formulierung der Naturgesetze gleichgültig ist.

Werner Heisenberg, 1934 [35, p.670f]

The **Copenhagen interpretation** is the interpretation of quantum mechanics first expressed in 1927 by Bohr and Heisenberg. Until 1970, it has been (in various variants) the almost generally accepted interpretation though there is no document defining it precisely; its contents was stated in varying ways depending on the occasion. One of the probable reasons is that it had sufficient definiteness to guide theory, experiment, and their relationship, and was at the same time sufficiently vague that it allowed each user to make sense of its paradoxical features in a personal, subjective way.

In our classification of interpretations of quantum mechanics, the Copenhagen interpretation belongs to type (I); the term 'knowledge' used first by HEISENBERG [32] was not understood in the subjective way used in (K) but as the objective (through thought experiments theoretically accessible) knowledge of what is real and in principle observable about the system, whether observed or not.

One important feature of the Copenhagen interpretation is the so-called **Heisenberg cut**, first described by HEISENBERG [33] and BOHR [8, pp.580,584] – the artificial splitting of the world into a quantum domain and a classical domain. VON NEUMANN [54, p.223f] showed that this cut can be placed fairly freely without affecting the main conclusions.

While adequate for microscopic systems, the concept of a necessary cut fails systematically for sufficiently large systems. For example, as all measurements are done within the solar system, it excludes treating the solar system as a quantum system (e.g. measuring the mass of the earth).

As mentioned already in Subsection 4.4, the thermal interpretation nowhere imposes a

cut between microscopic and macroscopic. It is not needed: A paper by JEON & YAFFE [38] derives the hydrodynamic equations from quantum field theory without assuming a Heisenberg cut. Only the thermal interpretation is (implicitly) invoked, which allows them to identify field expectations with the classical values of fields.

According to the thermal interpretation, classical physics appears gradually as systems become more macroscopic. In continuation of the discussion in Subsection 2.3 we call a quantum system whose relevant quantities have a negligible uncertainty a **classical system**. It is typically described by nonequilibrium thermodynamics, as deduced from quantum statistical mechanics; see Subsection 4.5 below. Thus a classical system is still quantum mechanical when modeled in full detail, but only the macroscopic variables modeled by statistical mechanics are deemed to be relevant. Thus the thermal interpretation leads to a gradual change from quantum to classical as the system gets larger and the uncertainty of the collection of relevant quantities decreases.

But the thermal interpretation realizes a modified version of the Heisenberg cut as the choice of relevant variables in the coarse-grained description, which defines the split between system and environment. According to the thermal interpretation, there is no sharp cut but a smooth fuzzy boundary, of the same kind as the boundary between the Earth's atmosphere and interplanetary space. The bigger one makes the instrument the more classical it becomes and the more accurate become the pointer positions. In place of deciding where to place the Heisenberg cut we now have to decide at which level of description the $O(N^{-1/2})$ corrections can be neglected. This is a decision just like the decision of whether or not to include into the classical description of a pendulum the surrounding air and the way it is suspended, or whether taking it into account with a damping term, or even neglecting that as well, is enough.

In quantum-classical approximations of a quantum system, the Heisenberg cut is explicitly modeled by allowing for both classical and quantum degrees of freedom, neglecting only irrelevant variables. As we have seen, the quantum-classical description naturally fits the thermal interpretation since q-expectations occur explicitly in the dynamics.

Die 'Bahn' entsteht erst dadurch, daß wir sie beobachten
Werner Heisenberg, 1927 [32, p.185]

Another puzzling feature of the Copenhagen interpretation is that an individual few particle system has **no definite properties** before it is observed. Taking the Copenhagen interpretation as an irreducible description of the nature of things leaves one puzzled how then the observing instrument can be informed about what it observes – being virtually nonexistent before the act of observation: In his 1927 paper famous for the uncertainty relation, Heisenberg asserted that the path (of a particle) is created only through the act of observation. Thus the observation creates the properties. But it must be created by something to be observed! The thermal interpretation gives the natural answer that this happens because the fields provide the information about what is there to cause the detectors to respond, so that something is observed: When measured, particles appear as detection events created by the detector and mediated by fields (cf. property (P) from Part I [50, Subsection 4.4]).

In photodetection, tradition takes the individual detection results too seriously and dogmatically¹³ interprets the random counting events as signals of single photons arriving, with all the spooky problems associated with this view. In contrast, the thermal interpretation treats it (as Stokes would have done it in 1852) as a very uncertain measurement of energy density. Then the particle aspect completely disappears. This is an advantage since, as we have seen in Subsection 3.3, it is difficult to specify – even informally – a particle picture at finite times in terms of the underlying relativistic quantum field description. How to do this with some degree of mathematical precision is an unsolved problem.

Fragen wir also nicht, wo ist ein Teilchen genau, sondern begnügen wir uns, zu wissen, daß es in einem bestimmten größeren Raumteil ist: dann verschwindet der Widerspruch zwischen Wellen und Corpusculartheorie.

Max Born, 1929 [12, p.116]

There is no doubt that while an atom is in an ion trap it has a definite but uncertain position. We know it is there and can check efficiently the duration of its presence. Indeed, in order to be able to do experiments with single atoms at all we need to know that they are there! In the Copenhagen interpretation, this knowledge was outside the quantum domain, on the classical side of the Heisenberg cut. The thermal interpretation preserves the reality of atoms being somewhere reasonably well localized while rejecting the idealization (made in classical point mechanics) that this position is given by an exact 3-dimensional real vector. This assumption leads to paradoxes both in classical electrodynamics and in certain quantum mechanics experiments. In the thermal interpretation it is avoided from the start, since all quantities come with their intrinsic uncertainty.

A "particle trace" on a photograph is also measurable. Tradition postulates that a corresponding particle existed that left this trace. But this statement is not experimentally refutable by any means, hence is a metaphysical assumption. Assuming it we can **infer** the uncertain position and momentum of a particle that was supposed to create it at the time of its creation. But we might also argue as in Part I [50, Section 4.4] and declare in analogy with the bullet experiment discussed there the tracks as a measure of impact quality, not associated with any particle!

GRASSI [29] and JEON & HEINZ [37, Section 5.3] (and many others) treat (in line with the thermal picture) interacting elementary "particles" not as particles but as quantum fluids; only their freeze-out in scattering experiments produces particle-like objects, in a way more or less analogous to how condensed droplets appear in saturated liquids. However, the quantization introduces a discrete element into the quantum numbers (and hence the number and distribution) of the resulting droplets, measurable as impact events or particle tracks.

Jede Ortsbestimmung reduziert also das Wellenpaket wieder auf seine ursprüngliche Größe

Werner Heisenberg, 1927 [32, p.186]

¹³There is no way to test this assumption empirically.

The state of the system after the observation must be an eigenstate of [the observable] α , since the result of a measurement of α for this state must be a certainty.

Paul Dirac, 1930 [23, p.49]

A third significant feature of the Copenhagen interpretation is the so-called **collapse** of the wave function upon measurement, introduced in 1927 by HEISENBERG [32]. Collapse to what is controversial. The authoritative 2007 book by SCHLOSSHAUER [65], takes the "jump into an eigenstate" postulated by Dirac in the above quote to be part of what he calls the "standard interpretation" of quantum mechanics. On the other hand, the older but also authoritative 1977 book by LANDAU & LIFSCHITZ [42] explicitly remarks in the discussion in Section 7 that the state after the measurement is in general not an eigenstate.

The collapse requirement contradicts the unitary evolution of pure states through the Schrödinger equation (which is the mode of change of a closed system, hence in the absence of a measurement), and depends on a not further detailed notion of measurement residing on the classical side of the cut. What happens to the state while the experiment is in progress but not complete is not specified. This makes the Copenhagen interpretation an incomplete description of the full temporal behavior of the state.

This incompleteness is a sign that we actually deal with a coarse-grained, reduced description. In such a reduced description, the description of the state of a particle is different before and after it passes a filter (polarizer, magnet, double slit, etc.). The new information that the particle passed the filter requires a different description analogous to that responsible for the use of classical conditional probability when additional information arrives. In the quantum case, this is modeled by the collapse of the wave function. Landau's general case is the one modeled by event-based filters (Subsection 2.5), while Dirac's situation is modeled by the special case where the filter operators R_k are the spectral projectors of an ideal measurement (Subsection 5.2).

In the thermal interpretation, collapse results from coarse-graining when the latter produces a reduced stochastic description in the form of a PDP (see Subsection 5.1). For example, we had seen in Subsection 4.6 that quantum-classical models may result from coarse-graining, and BONILLA & GUINEA [9] give an explicit quantum-classical model that exhibits chaos and collapse.

Understanding that collapse comes from coarse-graining is a similar insight as that friction comes from coarse graining, an insight familiar from classical mechanics treated in the Markov approximation for a few relevant quantities. In both cases, the insight bridges the difference in the dynamics of an isolated system and that on an open system obtained by hiding the environment, turning it into a source of stochastic events. The explanation by coarse graining is in both cases fully quantitative and consistent with experiment, hence has all the features a good scientific explanation should have.

5.4 The minimal interpretation

I reject the basic idea of contemporary statistical quantum theory, insofar as I do not believe that this fundamental concept will provide a useful basis for the whole of physics. [...]

One arrives at very implausible theoretical conceptions, if one attempts to maintain the thesis that the statistical quantum theory is in principle capable of producing a complete description of an individual physical system. On the other hand, those difficulties of theoretical interpretation disappear, if one views the quantum-mechanical description as the description of ensembles of systems. [...]

Within the framework of statistical quantum theory there is no such thing as a complete description of the individual system. [...]

it appears unavoidable to look elsewhere for a complete description of the individual system. [...]

I am rather firmly convinced that the development of theoretical physics will be of this type; but the path will be lengthy and difficult. [...]

the expectation that the adequate formulation of the universal laws involves the use of all conceptual elements which are necessary for a complete description, is more natural. [...]

If it should be possible to move forward to a complete description, it is likely that the laws would represent relations among all the conceptual elements of this description which, per se, have nothing to do with statistics.

Albert Einstein, 1949 [25]

The Statistical Interpretation, according to which a pure state (and hence also a general state) provides a description of certain statistical properties of an ensemble of similarly prepared systems. [...]

In general, quantum theory predicts nothing which is relevant to a single measurement (excluding strict conservation laws like those of charge, energy, or momentum), and the result of a calculation pertains directly to an ensemble of similar measurements. For example, a single scattering experiment consists in shooting a single particle at a target and measuring its angle of scatter. Quantum theory does not deal with such an experiment, but rather with the statistical distribution (the differential cross section) of the results of an ensemble of similar experiments.

Leslie Ballentine, 1970 [5, p.360f]

We can now define the scope of quantum theory: In a strict sense, quantum theory is a set of rules allowing the computation of probabilities for the outcomes of tests which follow specified preparations. [...] The above strict definition of quantum theory (a set of rules for computing the probabilities of macroscopic events) is not the way it is understood by most practicing physicists. They would rather say that quantum theory is used to compute the properties of microscopic objects, for example the energy-levels and cross-sections of atoms and nuclei. The theory can also explain some properties of bulk matter, such as the specific heat of solids or the electric conductivity of metals – whenever these macroscopic properties can be derived from those of the microscopic constituents. Despite this uncontested success, the epistemological meaning of quantum theory is fraught with controversy, perhaps because it is formulated in a language where familiar words are given unfamiliar meanings. Do these microscopic objects – electrons, photons, etc. – really exist, or are they only a convenient fiction introduced to help our reasoning, by supplying intuitive models in circumstances where ordinary intuition is useless?

Asher Peres, 2002 [57, p.13]

That quantum mechanical states should be interpreted statistically goes back to 1926. BORN [10, 11] had shown how the known statistical properties of scattering events is in some sense consistent with the deterministic Schrödinger equation and can be derived from it assuming a statistical interpretation of the wave function. This earned him later a Nobel prize. But in Born's view, particles existed (as beables) always in joint eigenstates of Hamiltonian and momentum that were modified discontinuously by random **quantum jumps**. In this way, the exact validity of the conservation laws of energy and momentum could be asserted.

Statistical interpretations in the precise sense (S) defined at the top of Section 5 have their beginnings with WEYL [75] and were discussed extensively by BALLENTINE [5], who contrasted it to the Copenhagen interpretation. In these statistical interpretations, a single (few or many particle) system has no state. Instead, the state is a property of the ensemble; one only talks about the prepared and observed properties of a population of experiments making up the ensemble. Equivalently, the preparation procedure (which defines the ensemble on which measurements are performed) has, or defines, a state.

The **minimal interpretation** is a statistical interpretation (S) augmented by the additional stipulation that quantum mechanics is completely silent about a single system, and hence says nothing at all about it.¹⁴

¹⁴True minimality is rare. EINSTEIN [25] finds only the minimal interpretation consistent, but takes this as a limitation of quantum mechanics and expects the existence of a deeper underlying deterministic description. BALLENTINE [5] is not minimal throughout (despite an attempt to be so), as he assumes (p.361) that definite positions exist: *"the Statistical Interpretation considers a particle to always be at some position in space, each position being realized with relative frequency $|\psi(r)|^2$ in an ensemble of similarly prepared experiments."* Even the most consequent book by PERES PERES makes an exception at the very end (p.424f): *"This would cause no conceptual difficulty with quantum theory if the Moon, the planets, the interstellar atoms, etc., had a well defined state ρ . However, I have insisted throughout this book that ρ is not a property of an individual system, but represents the procedure for preparing an ensemble of such systems. How shall we describe situations that have no preparer? [...] Thus, a macroscopic object effectively [...] mimics, with a good approximation, a statistical ensemble. [...] You must have noted the difference*

According to BALLENTINE [5, p.366], for a consistent statistical interpretation, the notion of **preparation** must be clearly distinguished from that of **measurement**: *”State preparation refers to any procedure which will yield a statistically reproducible ensemble of systems. The concept of state in quantum theory can be considered operationally as an abbreviation for a description of the state preparation procedure. Of course there may be more than one experimental procedure which yields the same statistical ensemble, i.e., the same state. An important special case (which is sometimes incorrectly identified with measurement) is a filtering operation, which ensures that if a system passes through the filter it must immediately afterward have a value of some particular observable within a restricted range of its eigenvalue spectrum. On the other hand, measurement of some quantity E for an individual system means an interaction between the system and a suitable apparatus, so that we may infer the value of R (within some finite limits of accuracy) which the system had immediately before the interaction (or the value of R which the system would have had if it had not interacted, allowing for the possibility that the interaction will disturb the system).”* The **filtering** mentioned replaces the collapse in the Copenhagen interpretation, and serves the same purpose. In the thermal interpretation, it is modeled by event-based filters (Subsection 2.5).

The thermal interpretation of the situation described is that the preparation defines a state of the quantum fields present in the description. Their interaction with the detector produces observable events, whose statistics measures properties of the fields. In principle, quantum tomography (see Subsection 2.5) can be used to calibrate sufficiently stationary unknown sources so that one can be sure which state they prepare in which setting. Knowing what was prepared and how to control it systematically, one can collect event statistics for new experimental settings and establish on the basis of the resulting experimental evidence a relation (1) between measurement results and properties of the system measured.

The single systems that allegedly travel from the source to the detector (but according to the minimal interpretation without any quantum properties) never enter the description, hence one cannot (and need not) say anything about these.

In a statistical interpretation, all statements claimed about single quantum systems are non-minimal. In particular, unlike the thermal interpretation, the minimal interpretation does not address the foundational problems posed by the ensembles of equilibrium thermodynamics (cf. Part II [51, Subsection 2.4]). Indeed, BALLENTINE [5, p.361] writes: *”The ensembles contemplated here are different in principle from those used in statistical thermodynamics, where we employ a, representative ensemble for calculations, but the result of a calculation may be compared with a measurement on a single system. [...] Because the ensemble in the statistical interpretation ”is not merely a representative or calculational device, but rather it can and must be realized experimentally, it does not inject into quantum theory the same conceptual problems posed in statistical thermodynamics.”*

between the present pragmatic approach and the dogmas held in the early chapters of this book.”

6 Conclusion

Nous tenons la mécanique des quanta pour une théorie complète, dont les hypothèses fondamentales physiques et mathématiques ne sont plus susceptibles de modification.

Max Born, Werner Heisenberg, 1927 [13, p.178]

According to the present quantum mechanics, the probability interpretation, the interpretation which was championed by Bohr, is the correct one. But still, Einstein did have a point. He believed that, as he put it, the good God does not play with dice. He believed that basically physics should be of a deterministic character.

And, I think it might turn out that ultimately Einstein will be proved right, because the present form of quantum mechanics should not be considered as the final form. [...] And I think that it is quite likely that at some future time we may get an improved quantum mechanics in which there will be a return to determinism and which will, therefore, justify the Einstein point of view.

Paul Dirac, 1975 [23]

According to the thermal interpretation, quantum physics is the basic framework for the description of objective reality (including everything reproducible studied in experimental physics), from the smallest to the largest scales. Classical descriptions are regarded as limiting cases when Planck's constant \hbar can be set to zero without significant loss of quality of the resulting models. The measurement of a Hermitian quantity A is regarded as giving an uncertain value approximating the q-expectation $\langle A \rangle$ rather than (as tradition wanted to have it) as an exact revelation of an eigenvalue of A . Single observations of microscopic systems are (except under special circumstances) very uncertain measurements only.

It seems that without having to introduce any change in the formal apparatus of quantum physics, the deterministic dynamics of the complete collection of q-expectations constructible from quantum fields, when restricted to the set of macroscopically relevant ones, already gives rise to all the stochastic features observed in practice.

The thermal interpretation

- is inspired by what physicists actually do rather than what they say. It is the interpretation that people actually work with in the applications (as contrasted to work on the foundations themselves), even when they pay lipservice to another interpretation.
- treats detection events as a statistical measurement of particle beam intensity.
- claims that the particle concept is only asymptotically valid, under conditions where particles are essentially free.
- claims that the unmodeled environment influences the results enough to cause all randomness in quantum physics.
- allows one to derive Born's rule for scattering and in the limit of ideal measurements; but in general, only part of Born's rule holds exactly: Whenever a quantity A with zero uncertainty is measured exactly, its value is an eigenvalue of A .

- has no explicit collapse – the latter emerges approximately in non-isolated subsystems.
- has no split between classical and quantum mechanics – the former emerges naturally as the macroscopic limit of the latter.
- has no split between classical and quantum mechanics – the former emerges naturally as the macroscopic limit of the latter;
- explains the peculiar features of the Copenhagen interpretation (lacking realism between measurements) and the minimal statistical interpretation (lacking realism for the single case) in the microscopic domain where these interpretations apply.
- paints a deterministic picture of quantum physics in which God does not play dice. It only seems so to us mortals because of our limited resolution capacity and since we have access to a limited part of the universe only.

In terms of the thermal interpretation, the measurement problem turns from a philosophical riddle into a scientific problem in the domain of quantum statistical mechanics, namely how the quantum dynamics correlates macroscopic readings from an instrument with properties of the state of a measured microscopic system.

While the present paper shows that in principle this is enough to resolve the riddles of quantum mechanics, a number of detailed questions remain open:

- The measurement principle (MP) from Subsection 2.1 demands that any instrument for measuring a quantity A has an uncertainty $\Delta a \geq \sigma_A$. It is an open problem how to prove this from the statistical mechanics of measurement models.
- The derivation of a piecewise deterministic stochastic process (PDP) by BREUER & PETRUCCIONE [16] suggests that, in general, that collapse in a single observed system – in the modern POVM version of the corresponding von Neumann postulate for quantum dynamics – is derivable from the unitary dynamics of a bigger system under the standard assumptions that go into the traditional derivations in classical statistical mechanics. It would be desirable to have a direct argument for this not dependent on a statistical approach.
- It should be possible to show in quantitative detail how position loses its parameter status and becomes uncertain when going from the relativistic quantum field description of a beam to a corresponding quantum mechanical description of a sequence of particles moving along the beam.
- It should be sufficient to explain from the dynamics of the universe the statistical features of scattering processes and the temporal instability of unobserved superpositions of pure states – as caused by the neglect of the environment.

The experimental evidence for the truth of all this is already there. Thus, unlike traditional interpretations, the thermal interpretation is an interpretation of quantum physics that is in principle refutable by theoretical arguments.

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