

On a realistic interpretation of quantum mechanics

Arnold Neumaier

Institut für Mathematik

Universität Wien

Strudlhofgasse 4

A-1090 Wien, Austria

email: neum@cma.univie.ac.at

WWW: <http://solon.cma.univie.ac.at/~neum/>

Abstract. The best mathematical arguments against a realistic interpretation of quantum mechanics – that gives definite but partially unknown values to all observables – are analysed and shown to be based on reasoning that is not compelling.

This opens the door for an interpretation that, while respecting the indeterministic nature of quantum mechanics, allows to speak of definite values for all observables at any time that are, however, only partially measurable.

The analysis also suggests new areas where the foundations of quantum theory need to be tested.

Keywords: realistic interpretation, quantum inseparability, probability, projector, measurement, spin, classical model, particle paths, double slit experiment

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1 Introduction

Quantum physics is a very successful theory for predicting nature. However, in spite of many attempts, a mathematically and philosophically convincing basis for the interpretation of quantum phenomena has not yet been found.

As stated recently by ZEILINGER [39], there are at least two levels of interpreting quantum mechanics: the statistical interpretation in the narrower sense introduced by BORN [8], on which there is almost complete consensus between physicists, and the interpretation of the meaning of the quantum mechanical concepts, where no agreement has been reached and all existing interpretations have been found wanting.

The ‘orthodox’ Copenhagen interpretation in terms of state reduction by external measurement loses credit because it becomes meaningless for the universe as a whole. There have been a number of proposals to change the structure of quantum mechanics, e.g., through pilot waves (BOHM [6]), consistent histories (e.g., OMNÈS [27]), event enhancement (BLANCHARD & JADCZYK [4]), or gravitational objective reduction (PENROSE [28]).

Perhaps the main reason why there is so little progress on the meaning of quantum mechanical concepts is that it is usually seen as thoroughly enmeshed with measurement problems. I believe that this is a mistake, and that more clarity can be obtained by separating the analysis of meaning from that of observability. The discussion thereby becomes more concise and clear.

Here I refer to *meaning* as a conceptual, logically consistent mathematical framework that allows one to speak unambiguously about *all* the terms used in the theory, in a way intuitively related to corresponding concepts of external reality. This part is often elegant and concise.

On the other hand, I refer to *observability* questions (or *measurement* problems) as the operational explanation of how to obtain quantitative values for the observables of a system, preferably within the framework of a well-defined mathematical theory. This part is usually messy, and it is the part that I propose to avoid until the other part is satisfactory. (This also saves me from entering a discussion about the meaning of words like ‘operational’ or ‘obtain’.)

For example, in classical real analysis (in contrast to non-standard analysis), infinitesimals are an ill-defined, meaningless concept, though they can be approximately simulated via the well-known δ, ε techniques, while random sequences are a well-defined, meaningful concept (see, e.g., CHAITIN [11]), though there is no constructive way of finding one, except approximately.

A historical case supporting the power of this view of separating the analysis of meaning from that of observability is the uncertainty principle. It was argued by HEISENBERG [16] in terms of observability, and later by ROBERTSON [30] in mathematical terms as a simple consequence of the canonical commutation relation $[p, q] = -i\hbar$. The mathematical argument takes only a few lines and is simple and compelling; the discussion of observability is complex but shows that the results of the mathematics cannot be in conflict with what can in principle be measured by experiment.

Further support for the positive effect of the separation of meaning from observability is given by Bell's inequality (BELL [2], CLAUSER & SHIMONY [12]), a purely algebraic statement whose clarity is compelling, and the subsequent verification of its violation through experiments by ASPECT [1].

Finally, it seems that measurement problems can be adequately analysed by generalized observables defined as positive operator valued (POV) measures; see, e.g., DAVIES [13], BUSCH et al. [10]. The gradual and approximate state reduction can be explained thermodynamically through dissipative interaction by the coupling to a macroscopic apparatus or heat bath; see, e.g., ZUREK [40], JOOS & ZEH [18], GHIRARDI, RIMINI & WEBER [15].

That obtaining information about a system and state reduction are not equivalent things is a consequence of the possibility of nondemolition measurements (BRAGINSKY et al. [9], KWIAT et al. [21]) that produce knowledge about a system but (almost) avoid state reduction. And since the measuring apparatus necessarily involves a huge number of particles, any satisfactory solution of measurement problems should not be a part of the foundational concepts but be obtained as their *consequence*.

In an attempt to separate the discussion about the conceptual meaning of quantum mechanics from that of experimental consistency, I searched the literature for concise statements of the basic difficulties in terms unrelated to measurement, but given in a clear mathematical context that allows one to analyse the logic of the problems without losing oneself in philosophical speculations.

The outcome was somewhat surprising: The best presented mathematical arguments turned out to have subtle flaws. This becomes apparent when one reduces them to arguments residing completely within the universally accepted statistical interpretation in the narrower sense.

Probably the weaknesses in the arguments escaped previous attention since the arguments were only used for philosophical discussion and justification of foundations, and never subjected to experiment. I therefore suggest below some specific things that one should try to test with suitable experiments.

In the following, I shall present my analysis of some arguments related to a realistic interpretation of quantum mechanics; *realistic* in the sense that it allows to speak of definite values for *all* observables at any time that are, however, only partially measurable.

The findings give rise to the hope that a consistent interpretation is possible that respects the stochastic nature of quantum mechanics, but is also realistic, with all the accompanying advantages for our intuition. In a companion paper (NEUMAIER [25]), a proposal for such a nonclassical but realistic interpretation will be presented.

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2 Feynman's argument

In his 1948 paper "Space-time approach to non-relativistic quantum mechanics" [14], Richard FEYNMAN gives on pp.368-369 the following argument.

"We define P_{ab} as the probability that if the measurement A gave the result a , then measurement B will give the result b . [...] denote by P_{abc} the probability [...] if A gives a , then B gives b , and C gives c . [...] If the events between a and b are independent of those between b and c , then

$$P_{abc} = P_{ab}P_{bc}. \quad (1)$$

This is true according to quantum mechanics when the statement that B is b is a complete specification of the state. In any event, we expect the relation

$$P_{ac} = \sum_b P_{abc}. \quad (2)$$

[...] Now, the essential difference between classical and quantum physics lies in Eq. (2). In classical mechanics it is always true. In quantum mechanics it is often false. [...] The classical law, obtained by combining (1) and (2),

$$P_{ac} = \sum_b P_{ab}P_{bc} \quad (4)$$

is replaced by

$$\phi_{ac} = \sum_b \phi_{ab}\phi_{bc}. \quad (5)$$

[for probability amplitudes ϕ_{ab} with $|\phi_{ab}|^2 = P_{ab}$] If (5) is correct, ordinarily (4) is incorrect. [...] Looking at probability from a frequency point of view (4) simply results from the statement that in each experiment giving a and c, B had some value. [...] Noting that *gzi5* replaces (4) only under the circumstance that we make no attempt to measure B, we are lead to say that the statement, ‘B had some value,’ may be meaningless whenever we make no attempt to measure B.”

Feynman’s argument sounds convincing but it has a flaw in the assumption of (1), the conditional independence of c from a, given b, for all states summed over in (2). This assumption is inconsistent with quantum mechanics.

Indeed, one of the characteristic features of quantum mechanics is its inseparability, the impossibility of splitting a quantum mechanical system into two that are independent. According to the Copenhagen interpretation (BOHR [7]), and verified abundantly by experiment, the *whole* space-time set-up must be specified to determine the correct outcome of an experiment. The contradiction obtained by Feynman in comparing (4) and (5) therefore need not have the consequence that B sometimes has no value, but serves more naturally as *a strong argument for quantum inseparability*.

The Markov property (1) does typically not even hold for consecutive events; due to the inherent noncommutativities, states ”remember” something about their whole history and ”anticipate” something about their whole future. To see this, pick a, b, c as observations of the same observable at increasing times (for memory) and at decreasing times (for anticipation). It should be an interesting project to find the conditions under which (and the accuracy to which) the Markov property for a completely observed time series of a sufficiently isolated system, lost in the quantum domain, is recovered in a thermodynamic limit of a large number of particles.

We now show in detail that (1) is inconsistent with quantum mechanics. According to the standard interpretation of probabilities as expectations of projection operators we have $P_{ab} = \langle a | \Pi_b | a \rangle$ and $P_{bc} = \langle b | \Pi_c | b \rangle$, where $\Pi_b = |b\rangle\langle b|$ is the projector to state b (and similarly for c). In order that P_{abc} makes sense in the standard interpretation we need $P_{abc} = \langle a | \Pi_{bc} | a \rangle$ with a projector Π_{bc} expressing the proposition $\Pi_b \& \Pi_c$ that B is in state b and C is in state c .

In orthodox quantum logics (BIRKHOFF & VON NEUMANN [3], SVOZIL [32]), one considers a projector as representing the subspace of eigenstates to the eigenvalue 1 of the projector and takes the logical operation $\&$ as subspace intersection. One finds that, whenever $\Pi_b \neq \Pi_c$, we have $\Pi_b \& \Pi_c = 0$ and thus $P_{abc} = 0$. This is different from (1), proving that (1) has nothing to do with quantum mechanics. And when $\Pi_b = \Pi_c$, we have $\Pi_b \& \Pi_c = \Pi_c$, hence

$P_{abc} = \langle a|\Pi_c|a\rangle = P_{ac}$, and (2) turns out to be *always* true in the quantum logic calculus, in contrast to what Feynman claimed.

On the other hand, the statement $P_{abc} = 0$ if $\Pi_b \neq \Pi_c$ seems somewhat counterintuitive, since if $\Pi_b \approx \Pi_c$ one might want to have $P_{abc} \approx P_{ac}$. (As pointed out by Karl Svozil, this observation has been made before by SZABO [33].) In view of the fact that, unless Π_b and Π_c commute, there is no simple *algebraic* expression relating Π_{bc} to Π_b and Π_c , one is not compelled to adhere to the rigid assignment $\Pi_b \& \Pi_c = 0$ and might prefer to consider $\Pi_b \& \Pi_c$ to be undefined if Π_b and Π_c do not commute, interpreting this as an indication that P_{abc} cannot be predicted (in principle) by the formalism of quantum theory (and thus may have any value).

In view of the fact that the counterintuitive abundance of zero projectors has further undesirable consequences to be discussed in Section 5, this would seem a more satisfactory solution of Feynman's argument. A new quantum logic consistent with this interpretation is discussed in NEUMAIER [25].

3 Wigner's claim

Similar discussions as those given by Feynman usually suffer from the same problem that the claimed probability cannot be written as the expectation of a quantum proposition, i.e., a projector. For example, in his 1976 lecture notes on the 'interpretation of quantum mechanics', WIGNER [37, p.288] writes,

[...] it remains essentially correct to say that the basic statement of quantum mechanics can be given in a formula as simple as (42).

In adapted notation, for a finite sequence $a, b, c, d \dots$ of results of measurements of A, B, C, D, \dots , it reads

$$\begin{aligned} P_{abcd\dots} &= \text{tr } \Pi_a \Pi_b \Pi_c \Pi_d \cdots \Pi_d \Pi_c \Pi_b / \text{tr } \Pi_a \\ &= \langle a | \Pi_b \Pi_c \Pi_d \cdots \Pi_d \Pi_c \Pi_b | a \rangle \quad \text{if } a \text{ is a pure state.} \end{aligned} \tag{42}$$

Now there is a problem with (42). It implies that repeating the measurement of A immediately after a first measurement of A gives the result of the first measurement with certainty. This is possible only if we assume that the measurements performed are ideal.

But (cf. Wigner's discussion in pp.283-284 of [37]; see also [18]) one cannot make an ideal measurement of an observable with a continuous spectrum; and the quantum mechanical analysis of the measurement process shows that an

ideal measurement of any quantity takes, strictly speaking, an infinite time. And even allowing for that, the only measurable observables would be the functions of scattering invariants (p.298). Hence formula (42) can apply to real situations only approximately.

Therefore, unlike Schroedinger's equation, (42) cannot be considered a 'basic statement' and must be banned from the foundations. It should rather be a consequence of more elementary, exact features of the theory.

Indeed, Wigner 'derives' (42) from first principles. The argument, given on pp.286-287, is identical to Feynman's, assuming (without comment) the Markov property that provides the independence that allows to multiply probabilities:

If the first observation [...] This probability is $|(a_\kappa, b_\lambda)|^2$. The probability that the next measurement [...] is then $|(b_\lambda, c_\mu)|^2$ and the probability of both outcomes [...] is $|(a_\kappa, b_\lambda)(b_\lambda, c_\mu)|^2$ [...].

Thus the formula, based on invalid reasoning, is suspect.

This suspicion is confirmed by realizing that the operator whose expectation is taken is generally not a projector, hence does not correspond to a 'proposition' in the traditional quantum logic calculus. But what sense should it make to talk about the probability of a statement that is not even logically well-formed?

It seems to me that the only formula for probabilities verified (abundantly) by experiments is the formula (see, e.g., equation (1.8) in Chapter 3 of DAVIES [13])

$$P(B \in E|\rho) = \text{tr } \rho B(E) \quad (6)$$

for the probability that, in a mixed state with the density matrix ρ , the generalized observable (POV measure) B has a value from a set E . For ordinary observables B and singleton sets $E = \{b\}$, the 'effect' $B(E)$ reduces to a projector Π_b . Equation (6) contains a special case of (42) only,

$$\begin{aligned} P_{ab} &= \text{tr } \Pi_a \Pi_b / \text{tr } \Pi_a \\ &= \langle a | \Pi_b | a \rangle = |\langle a | b \rangle|^2 \quad \text{if } \Pi_a = |a\rangle\langle a|, \langle a | a \rangle = 1. \end{aligned} \quad (7)$$

It is interesting to note that Wigner's formula (42) reappears as the basic formula in the theories of consistent histories (e.g., equation (4.3) in OMNÈS [27]). However, as mentioned on p.143 of [27], only the case of two reference times can be proved, and this is equivalent to (6) (with ρ replaced by $E_1 \rho E_1$, the mixed state obtained after the first state reduction).

However, we can give the probability (42) an alternative, correct meaning: If a is a pure state then (6) implies that P_{abcd} is the probability of measuring $\Pi_a\Pi_b\Pi_cD\Pi_c\Pi_b\Pi_a$ in that state! In general, therefore, Wigner's formula gives the probability of measuring in state a the last element in a finite sequence $A, \Pi_aB\Pi_a, \Pi_a\Pi_bC\Pi_b\Pi_a, \Pi_a\Pi_b\Pi_cD\Pi_c\Pi_b\Pi_a, \dots$, assuming that the measurements resulted in reductions of the intermediate states to the states represented by the projectors $\Pi_a, \Pi_b, \Pi_c, \dots$

To see how we can possibly interpret this in terms of the uncontroversial part of quantum mechanics, we consider a quantized relative of Laplace's demon. Suppose there were a quantum demon with the unusual capacity to 'see' every detail of a closed system, without interacting with the system. (The demon doesn't need to be physically realizable; this is just a fictional argument to make a vivid point.) The demon leaves the whole system (consisting of the measured system together with the measuring device) undisturbed and only interprets our claims on measurements and compares it with the dynamics it sees.

The demon notes the time intervals t, u, v, \dots between the successive measurements A, B, C, D, \dots . What does it see us measure? In the Heisenberg picture, with unlabelled operators at the time of the first measurement, the state of the complete system remains unchanged, and the measurements are those of $A, B(t), C(t+u), D(t+u+v), \dots$. If we use the Heisenberg dynamics and introduce the commuting operators $\Phi_a = \exp(itH/\hbar)$, $\Phi_b = \exp(iuH/\hbar)$, $\Phi_c = \exp(ivH/\hbar)$, we see that the demon sees us measure $A, \Phi_a^*B\Phi_a, \Phi_a^*\Phi_b^*C\Phi_b\Phi_a, \Phi_a^*\Phi_b^*\Phi_c^*D\Phi_c\Phi_b\Phi_a, \dots$. (This is quite different from what Wigner asserts in his equation (43), where he assumes an *additional* Heisenberg dynamics of the operators between the state-reducing measurements.)

Comparing with the above interpretation of (42), and noting that projectors are Hermitian, we see that the difference between the unproved formula (42) and the demon's objective description of our sequence of measurements is the replacement of the commuting unitary Φ 's in the complete system by the noncommuting idempotent Hermitian Π 's in the measured system alone.

Thus (42) is valid precisely to the extent that the time-dependent operators (in the Heisenberg picture), as calculated by the Heisenberg equation for the *complete* experimental setup at the time the next measurement is taken, can be approximated in the assumed (time-independent) state of the investigated *subsystem* by a two-sided multiplication with these projectors.

This shows clearly *the aim of a correct measurement theory*: It must exhibit broad sufficient conditions for a system that guarantee, for all observables to be measured, the validity of this approximation, and hence a complete

reduction of the wave packet.

The above analysis also suggests a new area where the foundations of quantum theory needs to be tested.

I'd like to challenge the adherents of (42) to devise and perform experiments testing validity, accuracy and limits of (42) when more than two (noncommuting) projectors are involved and the measurements are not ideal. If such experiments can be designed and performed at all, the findings should coincide with the interpretation in the Heisenberg picture just given; if not, (42) is meaningless since probabilities that cannot be observed are nonsense.

On the basis of such attempts it might also be possible to judge whether it is at all possible to perform experiments for measurements of probabilities that cannot be expressed as the expectation of a projector.

(For the realization of arbitrary discrete projectors, note that these can always be brought into the form UDU^* with diagonal projectors D and unitary U ; hence RECK et al. [29] applies.)

4 Schrödinger's argument

In his 1936 paper “Die gegenwärtige Situation in der Quantenmechanik” [31] (where the famous cat paradox appears), Erwin SCHRÖDINGER gives the following argument (pp. 156-157 of the English translation):

At first thought one might well attempt likewise to refer back the always uncertain statements of Q.M. to an ideal ensemble of states, of which a quite specific one applies in any concrete instance – but one does not know which one. That this won't work is shown by the one example of angular momentum, as one of many.

Imagine [...] the point M to be situated at various positions relative to O and fitted with various momentum vectors, and all these possibilities combined into an ideal ensemble. Then one can indeed choose these positions and vectors that in every case the product of vector length by length of normal OF [where F is the point closest to O on the line through M along the momentum vector] yields one or the other of the acceptable values – relative to the particular point O . But for an arbitrary different point O' , of course, unacceptable values occur. [...]

One could go on indefinitely with more examples. [...] Already for the single instant things go wrong. At no moment does there exist an ensemble of classical states of the model that squares with the totality of quantum mechanical

statements of this moment. [...] we saw that it is not possible smoothly to take over models and to ascribe, to the momentarily unknown or not exactly known variables, nonetheless determinate values, that we simply don't know.

The problem is indeed unsolvable if one insists on the existence of a *single* vector J of values determining all linear combinations $u \cdot J$ of the angular momentum. For example, in measuring the inner product $u \cdot J$ of the angular momentum J with four or more triplewise linearly independent unit vectors u , one obtains in each case values from the discrete spectrum of $u \cdot J$, and these spectra are inconsistent with a precise value of J . This is explained by Schrödinger in detail on p. 164 in the context of a simpler one-parameter example involving the observables $p^2 + a^2q^2$ where a is a parameter. Commenting on it, Schrödinger writes on p. 165:

Should one now think that because we are so ignorant about the relations among the variable-values held ready in one system, that none exists, that far-ranging arbitrary combination can occur? That would mean that such a system of "one degree of freedom" would need not merely two numbers for adequately describing it, as in classical mechanics, but rather many more, perhaps infinitely many.

Schrödinger dismisses this as not viable, but as I'll show now, there is a serious possibility that precisely this is the case, thus invalidating Schrödinger's conclusion that it is not possible to ascribe to the momentarily unknown or not exactly known variables determinate values. His conclusion is only valid if one ascribes to each observable vector K a determinate value that is to be used in the calculations for *all* $u \cdot K$.

However, as was observed already by VON NEUMANN [26, IV.1.E], one cannot, in general, combine the measuring recipes for two noncommuting observables to one for their sum, so that the sum of two observables is only implicitly characterized through the axioms. This should forbid the naive use of values for the components of K , say, to calculate the values of $u \cdot K$.

As I shall show now, one may consider each $u \cdot K$ as a (not necessarily classical) random variable in its own right, and determine their relationship for different u not by ordinary algebra but by the statistics derived from the standard quantum mechanical recipes.

For simplicity, I'll take for K in place of the angular momentum J the Pauli spin vector σ which shows precisely the same problems as J . For each unit vector $u \in \mathbb{R}^3$, the operator $u \cdot \sigma$ has the simple eigenvalues 1 and -1 ; thus these are the possible values of $u \cdot \sigma$. The projector to an eigenstate of $u \cdot \sigma$ corresponding to the eigenvalue 1 is $\Pi_u := \frac{1}{2}(1 + u \cdot \sigma)$, with $\text{tr } \Pi_u = 1$, and the corresponding projector for the eigenvalue -1 is simply Π_{-u} . For any unit vector $v \in \mathbb{R}^3$, the probability for $v \cdot \sigma$ having the value 1 when $u \cdot \sigma$ has

the value 1 is then, according to (7),

$$\begin{aligned}
 P_{uv} &= \text{tr } \Pi_u \Pi_v = \text{tr } (1 + u \cdot \sigma)(1 + v \cdot \sigma)/4 \\
 &= \text{tr } (1 + (u + v) \cdot \sigma + u \cdot v + i(u \times v) \cdot \sigma)/4 \\
 &= \frac{1}{2}(1 + u \cdot v),
 \end{aligned} \tag{8}$$

and the probability for $v \cdot \sigma$ having the value -1 when $u \cdot \sigma$ has the value 1 is therefore $P_{u,-v} = \frac{1}{2}(1 - u \cdot v)$, adding up to 1, as it should be. Moreover, $P_{u,-u} = 0$, as it should be. The probabilities depend continuously on u and v , in a very natural way.

One would get precisely the same probabilities if one had extremely fragile, classical spheres, painted white on one hemisphere and black on the other one, and each sphere would be destroyed after observing the color at a single point. One could still calculate probabilities for the colors of the other points, and any rotationally symmetric classical probability model produces precisely the same probabilities as we just calculated for the spin.

More details on such a classical model for electron spin (the so-called *Poincaré sphere* can be found in MITTELSTAEDT [23] and in KOCHEN & SPECKER [20]. The latter show that, under natural assumptions, such a classical model is restricted to two-state systems.

One such model is already enough to invalidate the cogency of Schrödinger's conclusion. However, in a companion paper [25], I'll show how one can reinterpret quantum mechanics in such a way that this model generalizes. In the generalization, the ordered set of quantum propositions is not embedded into a classical Boolean algebra but into a non-Boolean ordered set of 'selections', thus avoiding the no-go theorem of Kochen and Specker. Nevertheless, the new interpretation operates completely within the realm of classical logic.

To summarize, Schrödinger's argument demonstrates that, in quantum mechanics, one cannot calculate the values for linear combinations of noncommuting observables from the values of the observables themselves; but this is already obvious from the properties of the spectrum of operators. Only the values of functions of *commuting* observables (and, in particular, of functions of a single observable) can be predicted with certainty from the values of the observables themselves.

However, as was shown for the case of linear combinations of the Pauli spin matrices, his argument does not demonstrate that one cannot assign natural values to all observables that lead to natural and consistent classical probabilities.

The nature of quantum observations may put severe limits on what is observable through experiment and how these are combined to estimate the values

of other observables, but it does not seem to put restrictive limits on the joint values of noncommuting observables. Indeed, VON NEUMANN [26] shows in the same section mentioned above how to get estimates of joint probabilities of noncommuting quantities if sufficiently large ensembles are available.

5 Particle paths

It is widely believed that it is impossible to ascribe definite paths to moving quantum particles, except at special points where the position has been measured. The particle paths seen in a cloud chamber, say, can still be explained as an illusion created by correlation patterns among the atoms ionized by the particles; see MOTT [24]. No such analysis seems to be available, however, that would explain why we see macroscopic bound states (such as you or me) move along fairly definite paths.

The traditional context for discussing the inconsistency of definite particle paths is the double slit experiment (BOHR [7]), where it is claimed that particles passing a diaphragm containing the two slits cannot be said to have passed through one or the other slit, or – even worse – are said to have passed through both, in a way (superposition) that defies intuition.

Unfortunately, the discussion seems always to be connected to measurement questions, so that it is difficult to discern the mathematical content of the arguments that claim *nonexistence* of the path (in contrast to its *limited measurability* only, WOOTTERS & ZUREK [38]). However, I gathered some indirect evidence about particle paths by reading between the lines of some papers.

FEYNMAN [14], in laying ground work for his (nonrelativistic) path integral formalism, discusses on p. 371 the probability of particle paths as his first postulate:

If an ideal measurement is performed to determine whether a particle has a path lying in a region of space-time, then the probability that the result will be affirmative is the absolute square of a sum of complex contributions, one from each path in the region.

This postulate is used to show (among other things) that the most likely paths are those close to the path determined by the least action principle.

No comment is given on the precise mathematical meaning of the terms involved (except for the term “each”); thus we need to see whether we can interpret this in an orthodox way. In the spirit of the standard interpretation of quantum mechanics, we need to assign to each (let us say open and

bounded) region Ω of space-time a projector Π_Ω to the subspace \mathbb{H}_Ω of all wave functions belonging to states where the particle is with certainty in Ω . Unfortunately, particle positions at different times are represented by operators $x(t) = \exp(iHt)x(0)\exp(-iHt)$ that generally do not commute with each other, with the consequence that it is unlikely that they have common eigenvectors, which would be the elements of \mathbb{H}_Ω . Thus, it is very likely that, for most Ω , this subspace consists of zero only; and Π_Ω would vanish. This casts serious doubt on the applicability of the quantum logic recipe to defining the projector Π_Ω .

Perhaps it is even impossible to make consistent assignments of all projectors Π_Ω , in view of the fact that alternative attempts in consistent histories interpretations run into contradictions (KENT [19]). (But possibly these contradictions are due to the non-projector nature of these histories; cf. ISHAM [17].)

However, we can be more modest and only look at special sets of paths for which we may define proper projectors. For example, we may define the distance of two paths $\xi^k : [\alpha, \omega] \rightarrow \mathbb{R}^3$ ($k = 1, 2$) by

$$d(\xi^1, \xi^2) = \sqrt{\int_\alpha^\omega (\xi^1(t) - \xi^2(t))^2 dt}. \quad (9)$$

Then, for each specific path ξ , we can use the position operators $x(t) = \exp(iHt)x(0)\exp(-iHt)$ to define the observable $\Delta_\xi := d(x, \xi)$ that measures the distance of the particle path from ξ . Suitable projectors are now the projectors $\Pi_{\xi, \varepsilon}$ projecting to the subspace spanned by the wave functions corresponding to states in which Δ_ξ has with certainty a value not exceeding ε . Then, for a system in an arbitrary pure state a ,

$$P(d(x, \xi) \leq \varepsilon) = \langle a | \Pi_{\xi, \varepsilon} | a \rangle \quad (10)$$

is, according to the most orthodox interpretation of quantum mechanics, a well-defined quantum mechanical probability for the event that the particle path is, in the root mean square sense, within ε of the path ξ . Note that, in principle, this formula can be tested experimentally; and by varying ε , the complete distribution of $d(x, \xi)$ can be found. The natural path to be used for ξ is the expectation

$$\xi(t) = \langle a | x(t) | a \rangle \quad (11)$$

or a computable approximation to it (such as the classical path of the particle).

With formulas such as (9)–(11) available, it would be completely unreasonable to assume that a particle has no definite path. If it hadn't, what should

be the meaning of its expectation (11) and of the probability (10) of it being close to a given path?!

It remains to discuss how this finding can be reconciled with the double slit experiment. In an implementation of von Neumann's method mentioned in Section 4 to measure probabilities for two noncommuting observables, WOOTTERS & ZUREK [38] measure in a modified double-slit experiment both position and momentum of particles passing a slit:

If the measured momentum is positive, then we will guess that the photon passed through slit A; if it is negative, then we will guess that the photon passed through slit B. Clearly some of our guesses will be wrong – there are photons that have positive values of measured momentum even though their actual momentum was negative and they went through slit B.

[...] these two measurements cannot both be performed for the same photons, and so [the figures] cannot refer to the same experiment. Hence, in accord with the Copenhagen interpretation of quantum mechanics, there is no paradox. The complementarity principle does not prevent photons from behaving once as waves and once as particles. It only states that the same photon should not reveal this "split personality" in the same experiment. [...]

Despite the fact that we know with 99% certainty the paths of the photons, they still have strong wavelike properties. [...] we have presented a result which, although not paradoxical, was nevertheless surprising (that is, that one can make a fairly precise determination of the slit through which each photon comes with only a slight disturbance of the interference pattern).

The finding loses its surprise in the light of the above considerations that suggest existence of the path but with intrinsic limitations on its observability.

For the original double-slit experiment, the arguments generally agree on the observation that there is no way to predict through which slit a particle *will pass*. However, it is possible to compute with high confidence through which a particle *has passed* when it has been observed at the photographic plate. To do so, one only needs to compute the probabilities (10) for two typical paths ξ connecting each slit with the position where the particle was absorbed by the photographic plate. By comparing the likelihood ratios at a given significance level one will find an exceedingly high likelihood for most particles having passed through the slit closer to their recorded position on the photographic plate.

Thus, after the event, enough information is available to decide reasonably reliably on the particle path. One could also calculate the values of the actions of the paths through the two slits ending at the observed position,

and use the stationary phase approximation of the Feynman path integral to get (approximations to) the required probabilities. In any case, the statistical analysis is essentially similar to that used for all experiments where an event must be reconstructed from indirect measurable information together with knowledge about the way the experiment was prepared.

The key to getting more complete information about any microscopic system seems to be that one measures properties of the system at a number of different times and reconstructs by statistical methods the most likely values of the variables of interest, even of conjugate variables like position and momentum. An example for this are the particle tracks that are routinely reconstructed in high energy experiments (see, e.g. BOCK et al. [5]) that provide knowledge about both position and momentum of the particles involved.

One could ask about other situations where particle paths are counter to the established art of interpretation. A case in point is tunneling. If a particle follows in principle a definite path, how does it go through the potential wall in the tunnel effect?

The answer is that it doesn't go through the wall but over the mountain, so to speak. This happens already in classical stochastic systems, where one can calculate the transition rate in terms of the energy barrier. The equilibrium distribution is determined by the potential minima (and their Hessians); but the transition rate is extremely slow if the barrier is high. (The BSE disease is explained by such a metastable state.) In quantum mechanics, this is essentially the same effect, except that it can happen already at zero temperature. And all qualitative features are identical to those in statistical mechanics. For example (cf., e.g., MESSIAH [22, Section 3.1.6]), there is a dependence on the height of the barrier, and the tunnel effect is especially pronounced when the barrier is low and narrow. Thus, tunneling is a consequence of the stochastic nature of quantum mechanics, and one doesn't need a special interpretation in terms of quantum features.

6 Conclusion

The strongest arguments against realism available in the literature dissolved under a scrutinized analysis, using only that part of quantum mechanics for which there is almost universal agreement about its validity.

This opens the door for an interpretation that, while respecting the indeterministic nature of quantum mechanics, allows to speak of definite but only

partially measurable values for all observables at any time. This kind of realism is consistent with the intrinsic indeterminism required by Heisenberg's uncertainty relation if we distinguish carefully between *what is* and *what is measurable*.

Though not proved by the present investigations, it appears that, independent of the detailed description, a cautious realistic interpretation of quantum mechanics is in full accord with the generally accepted quantum mechanical formalism.

Here *realistic* means that in a completely specified state (not to be confused with the 'pure states' of quantum mechanics) *all* observables have definite values at all times that are, however, only partially measurable, according to the stochastic predictions of quantum mechanics.

And *cautious* means, that one has to take into account the following four restrictions:

1. All measurable and predictable probabilities are probabilities defined by orthogonal projectors (for sharp observables) or POV measures (for smeared observables) via equation (6).
2. For noncommuting projectors, the logical operation 'and' is not defined; the corresponding questions may be asked (and may have definite, i.e., logically consistent answers) but a precise answer cannot be found by experiments.
3. From the values of commuting observables one can deduce values of functions of these observables, but from the values of noncommuting observables one cannot even deduce the values of their linear combinations. Instead, functions of noncommuting observables must be considered as random variables on their own whose expectation values (and probability distributions) must be calculated from the general quantum mechanical formalism and not from a classical inference of measured linear combinations.
4. Due to quantum inseparability, the Markov property for a completely observed time series is lost. One only has a weaker quantum Markov property (without an intuitive interpretation for observed time series), valid in dissipative open systems (see, e.g., DAVIES [13].)

Note that what physicists actually measure and calculate in comparing theory and experiment, is in full agreement with restrictions 1–3. Separability (violating condition 4) is usually assumed between system and environment, but physicist are aware that this is an assumption that holds only approximately.

A constructive proposal for an interpretation respecting these restrictions will be given in a companion paper [25].

Appendix: Some final speculations

In this section I want to be more speculative, at the risk of being less precise, less cogent, and more vulnerable about some issues discussed. I shall mention some ideas and ‘conclusions’ that are related to the preceding analysis though far from being consequences of the rigorous observations of the preceding sections.

A realistic interpretation of quantum mechanics is independent of the measurement problem and in better accord with classical (say, 19th century) intuition. As we have seen, there seem to be no longer strong arguments against a cautious realistic interpretation of quantum mechanics.

A problem remaining, and one that obscured for a long time the underlying simplicity, is the lack of observable information on the quantum level. As SCHRÖDINGER [31] puts it (p.159 of the English translation), referring to the complementarity between position and momentum in the attempt to determine particle paths,

[...] the momentary statement content of the ψ -function is far from complete; it comprises only about 50 percent of a complete description.

However, unlike Schrödinger, I conclude that, while the remaining 50 percent will probably always remain unobservable, they may have a reality just as objective as the (ideally) observable 50 percent. The fact that in many cases we can choose which 50 percent of a complete description we want to observe underlines this conclusion.

The fact that both past and future boundary conditions are needed (and sufficient) to locate a particle path is consistent with the observation in Section 2 that states anticipate something about their whole future. It is also reminiscent of the action-at-a-distance formulation of classical electrodynamics by WHEELER & FEYNMAN [34, 35], where electromagnetic radiation is explained through interaction with particles in the future.

By observing the right 50%, namely past *and* future position boundary conditions about *one* of a pair of conjugate variables at both boundaries, we might be able, in principle, to reconstruct the complete intermediate picture as reliably as in the classical case, where one has the same option. But in the classical case one has the additional and, for us as subjects acting based on past information only, more useful option to predict the particle path from initial conditions only (position and momentum in the past).

This state of affairs can be summarized in the statement:

Physics essentially describes nature as if everything had already happened, and then expresses its laws as information about observed correlations. Since

some of the correlations involve time, it is possible to partially predict the future from the past, or the past from the future, or an intermediate situation from past and future observations.

This view of the world from the ‘end of times’ is usually referred to as the *block universe*. However, it seems to have escaped notice before that there is no operational way to distinguish between a universe that ‘happens’ and a block universe. This explains why questions such as the flow of time or free will cannot be discussed within the framework of physics.

Whether or not time flows, whether or not our will is free, the four-dimensional picture resulting from the course of nature, whether or not influenced by us, can (in a gedankenexperiment) be replayed, after everything has happened, like a movie. In the replay, everything is determined, and there is only the illusion of free will, just as we are used from the cinema. But the physics, expressed in the correlations between the parts of the movie, is identical to that in the original version.

The principle of physics, that it restricts attention to that which remains the same after everything happened, makes physics very powerful in that it allows us to investigate a past of billions of years and to anticipate a future of billions of years. But the same principle also generates its intrinsic limitations, that physics must be silent about everything that cannot be captured by this static, four-dimensional view of nature. This includes both questions such as “how does reality happen?”, and many of the subjects of most interest to people: freedom, purpose, and consciousness.

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